THE ASSOCIATION OF MATHEMATICS TEACHERS OF INDIA

AMTI – NMTC – 19th October, 2024 – INTER FINAL

Instructions:

1. Answer all questions. Each question carries 10 marks.
2. Elegant and innovative solutions will get extra marks.
3. Diagram and justification should be given wherever necessary.
4. Before start answering, fill in the FACE SLIP completely.
5. Your 'rough work' should be in the answer sheet itself.
6. Maximum time allowed is THREE hours.

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1. The numbers $1, 2, \ldots, 9$ are randomly arranged on a circle. Prove that there are three adjacent numbers whose sum is at least 16.

2. Find all quadruples (a, b, c, d) of positive integers, where $a \geq b \geq c \geq d$ such that
$$\left(1+\frac{1}{a}\right)\left(1+\frac{1}{b}\right)\left(1+\frac{1}{c}\right)\left(1+\frac{1}{d}\right)=5.$$

3. Let $P(x)$ be a polynomial with integer coefficients. If $P(101)P(97) = -16$, find the value of $P(101) + P(97)$.

4. Let $ABCD$ be a quadrilateral and let P be a point in its interior. Let K, L, M, N be the feet of the perpendiculars from P onto the lines AB, BC, CD, DA respectively. Let H_a, H_b, H_c, H_d be the orthocenters of the triangles AKN, BKL, CLM, DMN respectively. Prove that H_a, H_b, H_c, H_d are the vertices of a parallelogram.

5. Let M be the midpoint of the side BC of a triangle ABC. Point K on the segment AM satisfies $CK = AB$. Denote by L the intersection of CK and AB. Prove that the triangle AKL is isosceles.

6. If $a, b, c > 0$ such that $abc = 1$, prove that
$$\frac{1}{ab+a+2}+\frac{1}{bc+b+2}+\frac{1}{ca+c+2} \leq \frac{3}{4}$$

7. P is a regular polygon with 2025 vertices. The vertices are colored with colors red, green and blue such that any two adjacent vertices have different colors. Show that we can divide the polygon into triangles with non intersecting diagonals whose end points have different colors.

8. During a break, 128 children at a school sit in a circle to play a game. The teacher walks clockwise around the circle to hand out chocolates to the children according to the following scheme: he selects one child and hands over a chocolate to the child, then he skips the next child and gives a chocolate to the next one, then he skips 2 children and gives a chocolate to the next one, then he skips 3 children and so on. Show that after many rounds, each child will have received at least one chocolate.
If there were 129 children to start with, will the same conclusion hold?

THE ASSOCIATION OF MATHEMATICS TEACHERS OF INDIA

NMTC – 19TH October, 2024.

Junior Level – IX and X Grades – Final

Instructions:

1. Answer all questions. Each question carries 10 marks.
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1. Find the number of sets of 4 positive integers less than or equal to 25, such that the difference between any two elements in the set is at least 3.

2. Solve the system of equations:

$$\left(1+4^{2x-y}\right)5^{1-2x+y} = 1+2^{2x-y+1}$$

$$y^3+4x+1+\log(y^2+2x)=0$$

3. a, b, c, d, e, f are real numbers such that the polynomial

$$p(x) = x^8 - 4x^7 + 7x^6 + ax^5 + bx^4 + cx^3 + dx^2 + ex + f$$

factorises into eight linear factors $(x - x_i)$ with $x_i > 0$, for $i = 1, 2, 3, \ldots, 8$. Find the possible values of the constant f.

4. An acute angled triangle PQR is inscribed in a circle. Given $\angle P = 60^\circ$ and $\angle R > \angle Q$. Let H and I be the orthocentre and ncentre of ΔPQR respectively. Find the ratio of $\angle PHI$ to $\angle PQR$.

5. ABC is a triangle in which $\angle A$ is obtuse. AD, BE and CF are the altitudes from A, B and C respectively to BC, CA and AB.
A_1, B_1 and C_1 are arbitrary points on BC, CA and AB respectively. The circles on AA_1, BB_1 and CC_1 as diameters are drawn. Show that the lengths of the tangents from the orthocentre of ΔABC to these circles are equal.

6. Find all natural number solutions of the equation $3^x - 5^y = z^2$.

7. a, b, c are positive reals such that $ab + bc + ca = 3abc$. Show that $a^2b + b^2c + c^2a \geq 2(a+b+c) - 3$. When will the equality hold?

8. A regular polygon with 100 vertices are given. To each vertex, a natural number from the set { 1, 2, 3, … , 49 } is assigned. Prove that there are four vertices A, B, C, D such that if the numbers a, b, c, d are assigned to them respectively then $a + b = c + d$ and $ABCD$ is a parallelogram.

End of Question Paper

THE ASSOCIATION OF MATHEMATICS TEACHERS OF INDIA

NMTC – 19TH October, 2024.

Primary Level - V and VI Grades – Final

Instructions:

1. Answer all questions. Each question carries 10 marks.
2. Elegant and innovative solutions will get extra marks.
3. Diagram and justification should be given wherever necessary.
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5. Your 'rough work' should be in the answer sheet itself.
6. Maximum time allowed is THREE hours.

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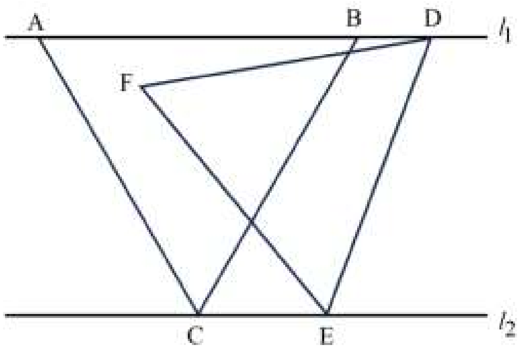
1. A 3-digit number of different digits is formed out of the digits 1, 3, 5 and 7; a single digit is left over.

If we multiply the 3-digit number by the left over single digit, find the greatest and smallest product that we can have.

2. Study the given figure:



ℓ_1 and ℓ_2 are two parallel lines.

Two equilateral triangles ABC and DEF are placed as shown in the figure, such that the angle between BC and FD is 39°.

Find the angle made by the side ED with the common perpendicular to these parallel lines.

3. Given:

$$A = 5 + \cfrac{1}{4 + \cfrac{1}{3 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{5}}}}}.$$

When simplified, A takes the form $\frac{p}{q}$ where p, q are positive integers with no common factors. Prove that $5\frac{1}{5} < A < 5\frac{2}{5}$.

4. There are two fractions f_1 and f_2, both in their lowest terms.

The numerator of f_2 is 3 more than the numerator of f_1.

The denominator of f_1 is 20 less than the denominator of f_2.

In f_1, if we subtract twice the numerator from the denominator, we get 3.

In f_2, if three times numerator is subtracted from the denominator, we get 2.

Find the value of $47\ f_2 - 27\ f_1$.

5. In the addition sum shown, A, B and C stand for different digits.

What is the value of the product ABC?

$$\begin{array}{r} A\ B\ C \\ A\ C\ C \\ +\ A\ B\ C \\ \hline 4\ 7\ 9 \end{array}$$

6. There are 3 positive numbers a, b and c.

The ratio of the first to the sum of the other two is 1 : 3.

The ratio of the third to the sum of the other two is 5 : 7.

Find the ratio of the second to the sum of the other two.

7. Find all 5-digit numbers of the form $34a5b$ where a, b are digits, such that they are divisible by 36.

8. For any natural number 'n' prove that $\frac{n}{3} + \frac{n^2}{2} + \frac{n^3}{6}$ is also a natural number.

End of Question Paper

THE ASSOCIATION OF MATHEMATICS TEACHERS OF INDIA
NMTC – 19TH October, 2024.
Sub junior Level - VII and VIII Grades – Final

Instructions:

1. Answer all questions. Each question carries 10 marks.
2. Elegant and innovative solutions will get extra marks.
3. Diagram and justification should be given wherever necessary.
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5. Your 'rough work' should be in the answer sheet itself.
6. Maximum time allowed is THREE hours.

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1. ℓ, m, n are positive integers such that
m divides $\ell+1$, n divides $m+1$ and ℓ divides $n+1$.
Find all possible triples (ℓ, m, n)

2. Consider 43 positive integers. Show that there exist two of them, say x and y, such that $x^2 - y^2$ is divisible by 100.

3. *ABCD* is a quadrilateral and *G*, *H* are the middle points of *AC* and *DB* respectively. If *CB*, *DA* are produced to meet at *E*, prove that the area of ΔEGH is equal to ¼ (area of quadrilateral *ABCD*).

4. *a*, *b*. *c* are three non-negative real numbers whose sum is not greater than ½. Show that

$$(1-a)(1-b)(1-c) \geq \tfrac{1}{2}$$

5. The vertical angle *A* of an isosceles triangle *ABC* is ⅓ of each of the base angles. Two points *M* and *N* are taken on *AB* and *AC* respectively so that $BM = BC = CN$. If *BN* and *CM* cut at *P*, show that $\angle MPN = \angle B$.

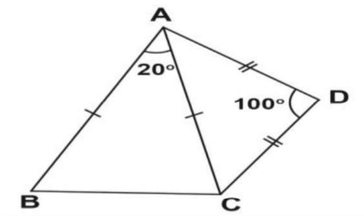
6. Solve for x, y, z :

$$12(x + y) = 5xy$$
$$24(y + z) = 7yz$$
$$8(z + x) = 3zx$$

7. Ten line segments are given. Each line segment has length between 1 cm and 55 cm. Prove that you can select three lines segments such that they form a triangle.

8. *ABC* and *DAC* are two isosceles triangles, with ∠BAC = 20° and ∠ADC = 100°.

Prove that *AB* = *BC* + *CD*.



End of Question Paper