ASSOCIATION OF MATHEMATICS TEACHERS OF INDIA

AMTI – NMTC – 2023 November – INTER – FINAL Class 11 & 12

Instructions:

1. **Answer all questions. Each question carries 10 marks.**
2. **Elegant and innovative solutions will get extra marks.**
3. **Diagrams and justification should be given wherever necessary.**
4. **Before starting to answer, fill in the FACE SLIP completely.**
5. **Your 'rough work' should be done in the answer sheet itself.**
6. **Maximum time allowed is THREE hours.**

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1. Prove that $\frac{29-5\sqrt{29}}{58}\left(\frac{7+\sqrt{29}}{2}\right)^{2023}+\frac{29+5\sqrt{29}}{58}\left(\frac{7-\sqrt{29}}{2}\right)^{2023}$ is an integer.

2. In a rectangle of area 12 are placed 16 polygons, each of area 1. Show that among these polygons there are at least two which overlap in a region of area at least $\frac{1}{30}$.

3. E and F are points on the sides CA and AB respectively of an equilateral triangle ABC such that EF is parallel to BC. G is the intersection point of medians in triangle AEF and M is a point on the segment BE. Prove that $\angle MGC = 60^\circ$, if and only if M is the mid-point of BE.

4. Let Q^+ denote the set of all positive rational numbers. Find all functions $f : Q^+ \to Q^+$ such that for all $x \in Q^+$

$$(a)\quad f(x+1) = f(x)+1$$

$$(b)\quad f(x^2) = ((f(x))^2$$

5. Find all values of n for which $1! + 2! + 3! + \ldots + n!$ is a perfect square.

6. Show that for all real numbers x, y the following inequality holds:

$$\sqrt{(x+1)^2+(y-1)^2}+\sqrt{(x-1)^2+(y+1)^2}+\sqrt{(x+2)^2+(y+2)^2} \geq 2\sqrt{2}+\sqrt{6}$$

7. Given 191 points in the plane such that no three are collinear. Let S be the largest set of triangles with vertices in the given points such that no two triangles in S have more than one vertex in common. Show that S contains at least 2023 triangles.

8. The polynomial $P(x)$ has integer coefficients and the coefficient of the highest degree term is 1. Also $P(x)$ satisfies

$$(x^3+3x^2+3x+2)(P(x-1) = (x^3-3x^2+3x-2)P(x)$$

For all real x. Find $P(\sqrt{2})$.

End of Question Paper

ASSOCIATION OF MATHEMATICS TEACHERS OF INDIA

AMTI – NMTC – 2023 November – JUNIOR – FINAL **Class 9 & 10**

Instructions:

1. Answer all questions. Each question carries 10 marks.
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6. Maximum time allowed is THREE hours.

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1. Find integers m, n such that the sum of their cubes is equal to the square of their sum.

2. PQR is an acute scalene triangle. The altitude PL and the bisector RK of $\angle QRP$ meet at H (L on QR and K on PQ). KM is the altitude of triangle PKR; it meets PL at N. The circumcircle of $\Delta\, NKR$ meets QR at S other than Q. Prove that SHK is an isosceles triangle.

3. Let a_i (i = 1,2,3,4,5,6) are reals. The polynomial
$$f(x) = a_1 + a_2x + a_3x^2 + a_4x^3 + a_5x^4 + a_6x^5 + 7x^6 - 4x^7 + x^8$$
can be factorized into linear factors $x - x_i$ where $i \in \{1,2,3,\ldots,8\}$.
Find the possible values of a_1.

4. There are n (an even number) bags. Each bag contains at least one apple and at most n apples. The total number of apples is $2n$. Prove that it is always possible to divide the bags into two parts such that the number of apples in each part is n.

5. a, b, c are positive reals satisfying
$$\frac{2}{5} \le c \le \text{minimum}\{a,b\};\ ac \ge \frac{4}{15} \text{ and } bc \ge \frac{1}{5}.$$
Find the maximum value of $\left(\frac{1}{a} + \frac{2}{b} + \frac{3}{c}\right)$.

6. The sum of the squares of four reals x, y, z, u is 1. Find the minimum value of the expression $E = (x - y)(y - z)(z - u)(u - x)$. Find also the values of x, y, z and u when this minimum occurs.

7. Let n be a positive integer; and $S(n)$ denote the sum of all digits in the decimal representation of n. A positive integer obtained by removing one or several digits from the right hand end of the decimal representation of n is called a *truncation* of n. The sum of all truncations of n is denoted as $T(n)$.

   Prove that $S(n) + 9T(n) = n$.

8. $ABCD$ is a cyclic quadrilateral. The midpoints of the diagonals AC and BD are respectively P and Q. If BD bisects the $\angle AQC$, then prove that AC will bisect $\angle BPD$.

End of Question Paper

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ASSOCIATION OF MATHEMATICS TEACHERS OF INDIA

AMTI – NMTC – 2023 November - PRIMARY – FINAL Class 5 & 6

Instructions:

1. Answer all questions. Each question carries 10 marks.
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4. Before starting to answer, fill in the FACE SLIP completely.
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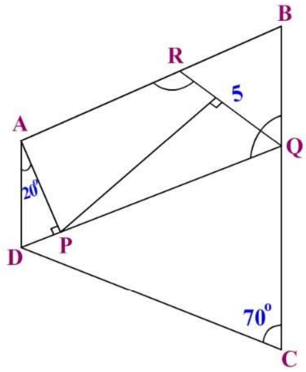
6. Maximum time allowed is THREE hours.

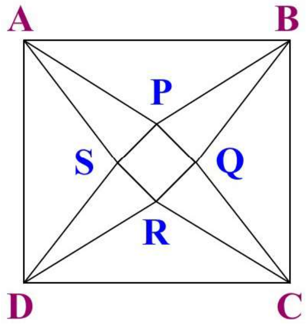
1. *ABCD* is an isosceles trapezium as shown in the figure, in which $AB = DC$; $\angle DAP = 20^o$; DP is perpendicular to AP; $\angle C = 70^o$; QR is the bisector of $\angle BQD$ and $PS \perp QR$. Calculate $\angle SPQ$ and $\angle SRA$. Justify each of the steps in calculation.



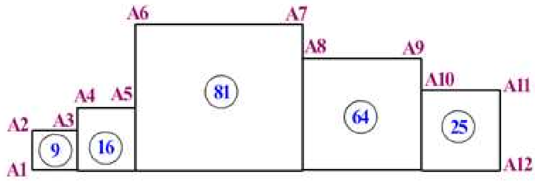
2. Ramanujan is a sixth grade student. His mathematics teacher gave a problem sheet in maths as home task for the Puja holidays. Ramanujan wants to complete it in 4 days and wants to enjoy the holidays for the remaining 6 days.
   On the first day, he worked out one-fifth the number of problems plus 12 more problems.
   On the second day, he worked out one-fourth the remaining problems plus 15 more problems.
   On the third day, he solved one-third of the remaining problems plus 20 more problems.
   The fourth day, he worked out successfully the remaining 60 problems and completed the work.
   Find the total number of problems given by the teacher and the number of problems solved by Ramanujan on *each* day.

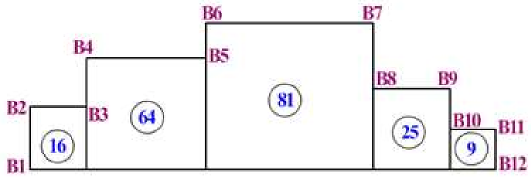
3. There are 4 cards and on each card a whole number is written. All the numbers are different from one another. Two girls of grade six, Deepa and Dilruba play a game.
   Deepa takes 3 cards at a time leaving a card behind. She multiplies the numbers and gets an answer. In the same way, again, she leaves one different card and selects the other three and multiplies the numbers. She got the answers 480, 560, 420 and 336.
   Now, Dilruba has to find the numbers in each card. Dilruba worked out and found the correct numbers.
   What are they? Work out systematically and find the numbers.

4. An angle is divided into 3 equal parts by two straight lines; such lines are called trisectors.
ABCD is a square. The lines (AP, AS) are trisectors of $\angle BAD$. Similarly, we have the trisectors (BP, BQ), (CQ, CR) and (DR, DS).
Prove that PQRS is a square.



5. Five squares of different dimensions are arranged in two ways as shown in the following diagrams. The numbers inside each square represents its area in square units.





Calculate the perimeter $A_1A_2A_3A_4A_5A_6A_7A_8A_9A_{10}A_{11}A_{12}A_1$ and the corresponding perimeter of figure 2. Are they same? If they are different, which is greater?

6. i) In a book, a problem on fractions is given as

$$\frac{1}{3\frac{1}{5}} - \frac{2\frac{1}{4}}{9} + \frac{3\frac{5}{8}}{\quad} + \frac{\frac{4}{7}}{4\frac{4}{7}}.$$

The denominator of the third term is not printed. The answer is given to be 2. What is the missing denominator? Let it be a.

ii) Simplify: $\dfrac{1}{3-\dfrac{1}{2-\dfrac{1}{(5/7)}}}$. Let the answer be of the form $\dfrac{p}{q}$ where p, q have no common factors. Let $b = \dfrac{p}{q}$.

iii) Find the value of $\left(\dfrac{1}{a^2}+b\right)$

7. a, b are two integers. Find all pairs a and b such that $\dfrac{1}{a}+\dfrac{1}{b}=\dfrac{1}{2}$. Arrive at your result logically.

8. A train starts from a station A and travels with constant speed up to 100 kms/hr. After some time, there appeared a problem in the engine and so the train proceeds with ¾ th of the original speed and arrives at Station B, late by 90 minutes. Had the problem in the engine occurred 60 kms further on, then the train would have reached 15 minutes sooner. Find the original speed of the train and distance between stations A and B.

End of Question Paper

ASSOCIATION OF MATHEMATICS TEACHERS OF INDIA

AMTI – NMTC – 2023 November – Sub-junior – FINAL Class 7 & 8

Instructions:

1. Answer all questions. Each question carries 10 marks.
2. Elegant and innovative solutions will get extra marks.
3. Diagrams and justification should be given wherever necessary.
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6. Maximum time allowed is THREE hours.

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1. If $b(a^2-bc)(1-ac)=a(b^2-ca)(1-bc)$ where $a\neq b$ and $abc\neq 0$, prove that

$$a+b+c=\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$$

2. a, b, c are three distinct positive integers. Show that among the numbers a^5b-ab^5, b^5c-bc^5, c^5a-ca^5 there must be one which is divisible by 8.

3. There are four points P, Q, R, S on a plane such that no three of them are collinear. Can the triangles PQR, PQS, PRS and QRS be such that at least one has an interior angle less than or equal to 45° ? If so, how? If not, why?

4. A straight line ℓ is drawn through the vertex C of an equilateral triangle ABC, wholly lying outside the triangle. AL, BM are drawn perpendiculars to the straight line ℓ. If N is the midpoint of AB, prove that Δ LMN is an equilateral triangle.

5. ABCD is a parallelogram. Through C, a straight line is drawn outside the parallelogram. AP, BQ and DR are drawn perpendicular to this line. Show that AP = BQ + DR. If the line through C cuts one side internally, then will the same result hold? If so prove it. If not, what is the corresponding result? Justify your answer.

6. m, n are non-negative real numbers whose sum is 1. Prove that the maximum and minimum values of $\dfrac{m^3+n^3}{m^2+n^2}$ are respectively 1 and ½ .

7. a) Solve for x: $\dfrac{x+5}{2018}+\dfrac{x+4}{2019}+\dfrac{x+3}{2020}+\dfrac{x+2}{2021}+\dfrac{x+1}{2022}+\dfrac{x}{2023}=-6$

   b) If $\dfrac{a^2+b^2}{725}=\dfrac{b^2+c^2}{149}=\dfrac{c^2+a^2}{674}$ *and* $a-c=18$, find the value of $(a+b+c)$.

8. If $a+b+c+d=0$, prove that $a^3+b^3+c^3+d^3=3(abc+bcd+cda+dab)$.

End of Qn. Paper.