ASSOCIATION OF MATHEMATICS TEACHERS OF INDIA
AMTI – NMTC - 2023 Jan. - PRIMARY – FINAL

Instructions:

1. Answer all questions. Each question carries 10 marks.
2. Elegant and innovative solutions will get extra marks.
3. Diagram and justification should be given wherever necessary.
4. Before start answering, fill in the FACE SLIP completely.
5. Your 'rough work' should be in the answer sheet itself.
6. Maximum time allowed is THREE hours.

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1. Three skilled workers Akbar, Baskar and Charles are employed by a person to do three different jobs. After completion of the work the total fee the person gave to the three workers is Rs 6000. It is found that Rs 400 more than $\frac{2}{5}$ of Akbar's share, Rs 200 more than $\frac{2}{7}$ of Baskar's share and Rs 100 more than $\frac{9}{17}$ of Charles' share are all equal. How much did each get?

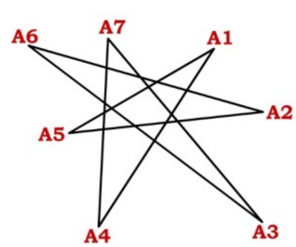
2. A teacher of a primary class asked his students to calculate

$$2\frac{3}{7} \text{ of } \frac{\left(13\frac{1}{2}-9\frac{2}{3}\right)}{\left(15\frac{1}{5}-11\frac{7}{30}\right)} = A$$

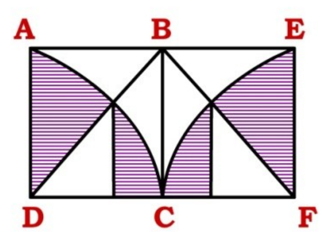
The teacher has 49A chocolates with him. He distributed equal number of chocolates (more than one chocolate) to each student of his class irrespective of whether the students got the correct answer or not. After the distribution the teacher is left with only one chocolate and he took it. Find the maximum strength of the class.

3. In a forest, Foxes always tell the truth and jackals always lie. When seen in poor light, they are indistinguishable. A person meets three of them A, B and C, in such a poor light. He asks A, "Are you a jackal?" Although A answers his question, he could not hear it clearly. B tells him that A denied being a Jackal and C says that A really is a jackal. Among the three how many are jackals?

4. Consider a natural number n. If n is less than 10 times the product of the digits, then n is called a *dwarf* number. Find the number of dwarf numbers between 10 and 200.



5. In the given figure, $A_1A_2A_3A_4A_5\ A_6A_7$ is a 7-pointed star. Find the value of $\angle A_1 + \angle A_2 + \angle A_3 + \angle A_4 + \angle A_5 + \angle A_6 + \angle A_7$



6. Two squares of side length 20 cm are joined together as in the diagram. With D, F as centres, quadrants are drawn. Taking π= 3.14, find the area of the shaded portion. Let A be the area in cm^2. Find A.

End of Qn. Paper

ASSOCIATION OF MATHEMATICS TEACHERS OF INDIA

AMTI – NMTC - 2023 Jan. – SUB-JUNIOR – FINAL

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1. A tray contains of 40 toffees. Jaya and Uma take in turn some toffees from the tray. Each time they are allowed to take 1, 2 or 3 toffees only. The person who gets the last toffee, wins. Jaya starts the game. Will she win? If so, how? If not, why?

2. Given $A = \left\{\dfrac{\frac{(a+b)^2+(a-b)^2}{b-a}-(a+b)}{\frac{1}{b-a}-\frac{1}{a+b}}\right\} \div \left\{\dfrac{(a+b)^3+(b-a)^3}{(a+b)^2+(a-b)^2}\right\}$

$$B = \left\{\frac{c-b}{(a-b)(a-c)} - \frac{c-a}{(b-c)(b-a)} + \frac{b-a}{(c-a)(a-b)}\right\} \div \left\{\frac{2(a^2+b^2+c^2-ab-bc-ca)}{(a-b)(b-c)(c-a)}\right\}$$

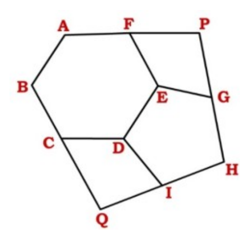
If $a = 2022$, $b = 2023$, find $(A+B)$.

3. There are square papers of areas 1, 2, 3, . . . square millimetres. Asif started colouring them with red and green paint. He painted in red the papers whose areas can be written as the sum of two composite numbers and the rest in green. How many papers are painted green and what are their areas?

4. Find natural numbers *a*, *b*, *c* such that their sum is 6, sum of their squares is 14 and the sum of the products of *a* and *c*, and *b* and *c* is equal to the square of one more than the product of a and b.

5. The volumes of three cubic containers C_1, C_2, and C_3 are in the ration 1: 8: 27. The amount of water in them are in the ratio 1:2:3. Water is poured from C_1 to C_2 and from C_2 to C_3 then the water level in all the containers is the same. Now $128\frac{4}{7}$ litres of water is poured out from C_3 to C_2 after which a certain amount is poured from C_2 to C_1 so that the depth of water in C_1 becomes twice that in C_2. This results in the amount of water in C_1 100 litres less than the original amount. How much water did each container contain originally?

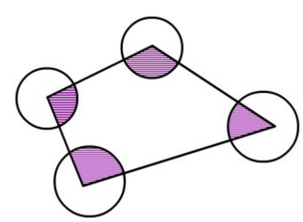
6. In the adjoining figure, *ABCDEF* and *DEGHI* are respectively a regular hexagon and a regular pentagon. *P* is the intersection of AF and *HG* and similarly *Q*. The bisector of $\angle FPG$ cuts *AB* produced at *R*. Prove that $\angle Q = 8\angle R$.



7. *PQR* is an equilateral triangle. *S* is any point inside the triangle. *SA, SB, SC* are respectively drawn perpendiculars to *PR, RQ and PQ*. Find the ratio of

$$\frac{SA+SB+SC}{QA+RB+PC}.$$

8. In the adjoining figure, four equal circles of radius 7 cm each are drawn with centres at the four vertices of a quadrilateral. Find the total area of the shaded sectors.



End of Question Paper

ASSOCIATION OF MATHEMATICS TEACHERS OF INDIA
AMTI – NMTC - 2023 Jan. – JUNIOR – FINAL

Instructions:

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1. x, y, z are positive reals and $(x + y + z)^3 = 32xyz$. Find the numerical limits between which the expression $\frac{x^4+y^4+z^4}{(x+y+z)^4}$ lies?

2. $A_1A_2A_3A_4A_5A_6A_7$ is a *regular* heptagon. Prove that

$$\frac{{A_1A_4}^3}{{A_1A_2}^3} - \frac{A_1A_7+2A_1A_6}{A_1A_5-A_1A_3} = 1$$

3. ABCD is a *square* whose side is 1 unit. Let n be an arbitrary natural number. A figure is drawn inside the square consisting of only line segments, having a total length greater than $2n$. (This figure can have many pieces of single line segments intersecting or non-intersecting). Prove that for some straight-line L which is parallel to a side of the square must cross the figure at least $(n+1)$ times.

4. m is a *natural* number. If $(2m+1)$ and $(3m+1)$ are perfect squares, then prove that m is divisible by 40.

5. Given 69 distinct *positive* integers not exceeding 100, prove that one can choose four of them a, b, c, d such that $a < b < c$ and $a+b+c = d$. Is this statement true for 68?

6. m, n are integers *such* that $n^2(m^2 + 1) + m^2(n^2 + 16) = 448$. Find all possible ordered pairs (m, n).

7. BD is the *bisector* of $\angle ABC$ of triangle ABC. The circumcircles of triangle BCD and triangle ABD cut AB and BC at E and F respectively. Show that $AE = CF$.

8. ***a*** is a two-digit number. ***b*** is a three-digit number. ***a*** increased by ***b*** percent is equal to ***b*** decreased by ***a*** percent. Find all possible ordered pairs (***a***, ***b***).

End of Question Paper

ASSOCIATION OF MATHEMATICS TEACHERS OF INDIA
AMTI – NMTC - 2023 Jan. – INTER – FINAL

Instructions:

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6. Maximum time allowed is THREE hours.

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1. The positive rational numbers are labelled in a sequence as follows:

$$\frac{1}{1},\frac{2}{1},\frac{1}{2},\frac{3}{1},\frac{2}{2},\frac{1}{3},\frac{4}{1},\frac{3}{2},\frac{2}{3},\frac{1}{4},\ldots\ldots$$

For example, $\frac{1}{2}$ occurs at position 3 for the first time and $\frac{2}{3}$ occurs at position 9 for the first time.

(a) Find the positions of the first five occurrences of $\frac{1}{2}$.

(b) At what position does the rational $\frac{22}{23}$ occur for the first time in the list?

2. Find all integers *x, y, z* satisfying the equations

$$(x+20)(y+12)(z-4) = 3000$$
$$(x+20)(y-12)(z+4) = 3000$$
$$(x-20)(y+12)(z+4) = 3000$$

3. Let ABC be an acute angled triangle with $\angle A = 60^o$ and $AB > AC$. Let I be the incentre and D, E, F be the mid points of the sides BC, CA, AB respectively.

(a) If H is the orthocentre of the triangle ABC, prove that $2\angle AHI = 3\angle B$.

(b) If M is the mid-point of AI, prove that M lies on the circumcircle of triangle DEF.

4. A line through a vertex of a triangle is called a *dividend* if it cuts the triangle into two triangles with equal perimeter. Let *ABC* be a triangle. Show that the dividends of *ABC* through the vertices *A, B, C* are concurrent.

5. Find all polynomials $P(x)$ with real coefficients such that $P(0) = -729$ and $(x-27)P(3x) = 27(x-1)P(x)$

6. There are 2023 people attending a conference. In any set of three people in the conference, either all the three know each other or exactly two among them know each other. Show that we can find a group of 1012 people at the conference who all know each other.

End of Question Paper