# THE ASSOCIATION OF MATHEMATICS TEACHERS OF INDIA

## Screening Test – Ramanujan Contest

NMTC at INTER LEVEL – XI & XII Standards

Saturday, 1st September, 2018

**Note:**

1. Fill in the response sheet with your Name, Class and the institution through which you appear in the specified places.
2. Diagrams are only visual aids; they are NOT drawn to scale.
3. You are free to do rough work on separate sheets.
4. Duration of the test: 2 pm to 4 pm (2 hours).

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**PART – A**

**Note**

- Only one of the choices $A, B, C, D$ is correct for each question. Shade the alphabet of your choice in the response sheet. If you have any doubt in the method of answering, seek the guidance of the supervisor.
- For each correct response you get 1 mark. **For each incorrect response you lose $\frac{1}{2}$ mark.**

1. In the addition shown, each of the letters $T, H, I, S$ represents a non zero digit. What is $T + H + I + S$?

$$\begin{array}{cccc} T & H & I & S \\ & & I & S \\ \hline 2 & 0 & 1 & 8 \end{array}$$

A. 34 B. 32 C. 24 D. 22

2. We have four sets $S_1, S_2, S_3, S_4$ each containing a number of parallel lines. The set $S_i$ contains $i+1$ parallel lines, $i = 1, 2, 3, 4$. A line in $S_i$ is not parallel to lines in $S_j$ when $i \neq j$. In how many points do these lines intersect?

A. 54 B. 63 C. 71 D. 95

3. An oil tanker is 100 km due north of a cruise liner. The tanker sails Southeast at a speed of 20 kilometres per hour and the liner sails Northwest at a speed of 10 kilometres per hour. What is the shortest distance between the two boats during the subsequent motion?

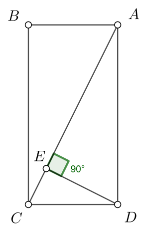
A. $50\sqrt{2}$ km B. 60 km C. 80 km D. 100 km

4. Volume $A$ equals one fourth of the sum of the volumes $B$ and $C$, while volume $B$ equals one sixth of the sum of the volumes $A$ and $C$. The ratio of volume $C$ to the sum of volumes of $A$ and $B$ is

A. $2:3$ B. $9:10$ C. $7:12$ D. $12:23$

5. In the *ninety-nine* shop every item costs some whole number of rupees plus 99 paise. Rhea spent sixty five rupees and seventy six paise in buying some items from the shop. How many items did she buy?

A. 23 B. 24 C. 65 D. 66




6. The diagram shows a rectangle $ABCD$ where $AB : AD = 1 : 2$. Point $E$ on $AC$ is such that $DE$ is perpendicular to $AC$. What is the ratio of the area of the triangle $DCE$ to the rectangle $ABCD$?

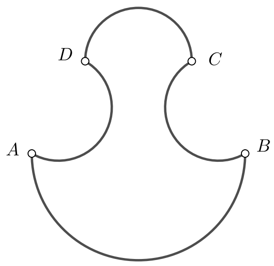
   A. $1 : 4\sqrt{2}$  B. $1 : 6$  C. $1 : 8$  D. $1 : 10$

7. The numbers $2, 3, 12, 14, 15, 20, 21$ may be divided into two sets so that the product of the numbers in each set is the same. What is this product?

   A. 420  B. 1260  C. 2520  D. 6720

8. $ABCD$ is a trapezium with $AD = DC = CB = 10$ units and $AB = 22$ units. Semi circles are drawn as shown in the figure. The area of the region bounded by these semi circles in square units is

   A. $128 + 48\pi$  B. $128 + 24\pi$  C. $116 + 48\pi$  D. $116 + 24\pi$




9. Consider the number of ways in which five girls and five boys sit in ten seats that are equally spaced around a circle. The proportion of the seating arrangements in which no two girls sit at the ends of a diameter is

   A. $\frac{1}{2}$  B. $\frac{8}{63}$  C. $\frac{55}{63}$  D. None of the above

10. Let $A = 1^{-4} + 2^{-4} + 3^{-4} + \cdots$, the sum of reciprocals of fourth powers of integers and $B = 1^{-4} + 3^{-4} + 5^{-4} + \cdots$, the sum of reciprocals of fourth powers of odd positive integers. The value of $A/B$ as a fraction is

    A. $\frac{16}{15}$  B. $\frac{32}{31}$  C. $\frac{64}{63}$  D. $\frac{128}{127}$

11. The number $5^{(6^7)}$ is written on the board (in base 10). Gia takes two of the digits at a time, erases them but appends the sum of those digits at the end. She repeats this till she ends up with one digit on the board. What is the digit that remains on the board?

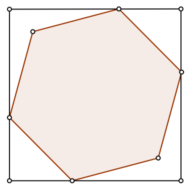
    A. 1  B. 5  C. 6  D. 7

12. Seven points are marked on the circumference of a circle and all pairs of points are joined by straight lines. No three of these lines have a common point and any two intersect at a point inside the circle. Into how many regions is the interior of the circle divided by these lines?

    A. 64  B. 63  C. 57  D. 56

13. The diagram below shows a regular hexagon with side length 1, inscribed in a square. Two of the vertices lie on the diagonal of the square and the remaining vertices lie on its sides. What is the area of the square?

A. $\frac{7}{2}$ B. 4 C. $2+\sqrt{3}$ D. $3+\sqrt{2}$



14. $AB$ is a diameter of a semicircle of center $O$. $C$ is the midpoint of the arc $AB$. $AC$ and the tangent at $B$ to the semicircle meet at $P$. $D$ is the midpoint of $BP$. If $ACDO$ is a parallelogram and $\angle PAD = \theta$, then $\sin\theta$ is

A. $\frac{1}{\sqrt{5}}$ B. $\frac{1}{\sqrt{10}}$ C. $\frac{2}{\sqrt{10}}$ D. $\frac{3}{\sqrt{10}}$

15. The real valued function $f(x)$ satisfies the equation $2f(1-x)+1 = xf(x)$ for all $x$. Then $(x^2-x+4)f(x)$ equals

A. $x-1$ B. $x$ C. $x+1$ D. $x-3$

## PART – B

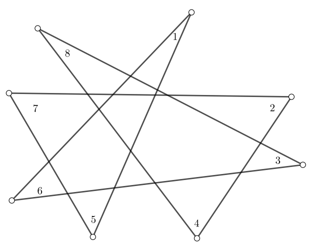
**Note**

- Write the correct answer in the space provided in the response sheet.
- For each correct response you get 1 mark. **For each incorrect response you lose $\frac{1}{4}$ mark.**

16. The number of ways in which 26 identical chocolates be distributed between Amy, Bob, Cathy and Daniel so that each receives at least one chocolate and Amy receives more chocolates than Bob is ______.

17. A set $S$ contains 11 numbers. The average of the numbers in $S$ is 302. The average of the six smallest numbers of $S$ is 100 and the average of the six largest of the numbers is 300. What is the median of the numbers in $S$ ______.

18. The sum of the angles $1,2,3,4,5,6,7,8$ in degrees shown in the following figure is ______.




19. The number of positive integers less than 2018 that are divisible by 6 but are not divisible by at least one of the numbers 4 or 9 is ______.

20. If

$$x(x+1)(x+2)\cdots(x+23) = \sum_{n=1}^{24} a_n x^n$$

the number of coefficients $a_n$ that are multiples of 3 is ———.

21. A square is cut into 37 squares of which 36 have area 1 square cms . The length of the side of the original square is ———.

22. There are 4 coins in a row and all are showing heads to start with. The coins can be flipped with the following rules:

    (a) The fourth coin (from the left) can be flipped any time

    (b) An intermediate coin can be changed to tail only if its immediate neighbor on the right is heads and all other coins (if any) to its right are tails.

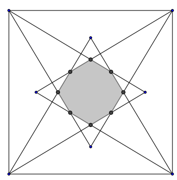
    (c) Only one coin can be flipped in one step.

    The minimum number of steps required to bring all coins to show tails is ———.

23. A poet met a tortoise sitting under a tree. When the tortoise was the poet's age, the poet was only a quarter of his current age. When the tree was the tortoise's age, the tortoise was only a seventh of its current age. If all the ages are in whole number of years, and the sum of their ages is now 264, the age of the tree in years is ———.

24. The sum of all real values of $x$ satisfying $\left(x + \frac{1}{x} - 17\right)^2 = x + \frac{1}{x} + 17$ is ———.

25. On the inside of a square with side length 6, construct four congruent isosceles triangles each with base 6 and height 5, and each having one side coinciding with a different side of the square. The area of the octagonal region common to the interiors of all four triangles is ———.



26. In a triangle with integer side lengths, one side is thrice the other. The third side is 15 cm. The greatest possible perimeter of the triangle is (in cm) ———.

27. A cube has edge length $x$ (an integer). Three faces meeting at a corner are painted blue. The cube is then cut into smaller cubes of unit length. If exactly 343 of these cubes have no faces painted blue, then the value of $x$ is ———.

28. If $f(x) = ax^4 - bx^2 + x + 5$ and $f(3) = 8$ , the value of $f(-3)$ is ———.

29. Archana has to choose a three-digit code for her bike lock. The digits can be chosen from 1 to 9. To help her remember them, she decides to choose three different digits in increasing order, for example 278. The number of such codes she can choose is ———.

30. Let $S$ be a set of five different positive integers, the largest of which is $n$ . It is impossible to construct a quadrilateral with non-zero area, whose side-lengths are all distinct elements of $S$ . The smallest possible value of $n$ is ———.