Success begins with you ...-

Note -
Elegant and novel solution will get extra Credits
Diagrams and explanation should be given wherever necessary.
Rough work should be shown in the answer copy itself.

1. Find the number of numbers coprime to and less than 2012. Find their sum. Find also the quotient when this sum is divided by 2012. (Information: 503 is a prime).

2. Composite twins are defined below:

(a) Odd composite twin: let a and a + 2 be two odd composite numbers. If (a - 2) and (a + 4) are primes then (a, a + 2) is called an "odd composite twin".

(b) Even composite twin: Let 6 and b + 2 two even numbers (b > 2). If (b - 1) and (6 + 3) are primes then (b, b + 2) is called an even composite twin.

List all composite twins less than or equal to 100.

3. ABCD is a rectangle. The sides are extended and the external angles are bisected and the bisectors are produced in both ways to form a quadrilateral. Prove that the quadrilateral is a square.

4. (a) A single digit natural number is increased by 10. The obtained number is now increased by the same percentage as in the first increase. The result is 72. Find the original single digit number.

(b) After two price reductions by one and the same percent the price of an article is reduced from Rs. 250 to Rs. 160. By how much percent was the price reduced each time. Write detailed steps.

5. If a finite straight line segment is divided into two parts so that the rectangle contained by the whole and first part is equal to the square on the other part, prove that the square described on one of the diagonals of the rectangle contained by the whole and the first part is three times the square on the other part.

6. abcde is a five digit number. Show that abcde is divisible by 7 if and only if the number abcd — (2 x e) is divisible by 7.

7. If $a^2x^3 + b^2y^3 + c^2z^3 = p^5, ax^2 = by^2 = cz^2$

and $\dfrac{1}{x} + \dfrac{1}{y} + \dfrac{1}{z} = \dfrac{1}{p}$ find $\sqrt{a} + \sqrt{b} + \sqrt{c}$ only in terms of p.

8. Take any natural number. Multiply it with the next two natural numbers. Take another natural number different from the first and do the same as before. Subtract one result from the other to get a positive difference and divide the difference obtained by the positive difference of the original numbers. Add to the quotient the product of the original numbers. Prove that the final result is the product of some number by the number next above it.