

THE ASSOCIATION OF MATHEMATICS TEACHERS OF INDIA

BHASKARA CONTEST – FINAL – JUNIOR

Classes IX & X

Saturday, 2nd October, 2016

Instructions:

1. Answer as many questions as possible.
2. Elegant and novel solutions will get extra credits.
3. Diagrams and explanations should be given wherever necessary.
4. Fill in FACE SLIP and your rough working should be in the answer book.
5. Maximum time allowed is THREE hours.
6. All questions carry equal marks.

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1. (a) If a, b, c are positive reals and $a+b+c = 50$ and $3a+b-c = 70$. If $x = 5a+4b+2c$, find the range of values of x .

- (b) The sides a, b, c of a triangle ABC satisfy the equation

$$a^2 + 2b^2 + 2016c^2 - 3ab - 4033bc + 2017ac = 0$$

Prove that b is the arithmetic mean of a, c .

2. In an isosceles triangle ABC , $AB = AC$. The bisector AD of $\angle A$ meets the side BC at D . The line perpendicular to AD through D meets AB at F and AC produced at E . Perpendiculars from B and D to AC are respectively BM and DN . If $AE = 2016$ units, find the length MN .

3. (a) Two circles with centres at P and Q and radii $\sqrt{2}$ and 1 respectively intersect each other at A and D and $PQ = 2$ units. Chord AC is drawn to the bigger circle to cut it at C and the smaller circle at B such that B is the midpoint of AC . Find the length of AC .

- (b) Find the greatest common divisor of the numbers $n^2 - n$, $n = 3, 5, 7, 9, \dots$

4. (a) A book contained problems in Algebra, Geometry and Number theory. Mahadevan solved some of them. After checking the answers, he found that he answered correctly 50% problems in Algebra, 70% in Geometry and 80% in Number theory. He further found that he solved correctly 62% of problems in Algebra and Number theory put together, 74% questions in Geometry and Number theory altogether. What is the percentage of correctly answered questions in all the three subjects?

- (b) Find all pairs of positive integers (a, b) such that $a^b - b^a = 3$.

5. a, b, c are positive real numbers. Find the minimum value of

$$\frac{a+3c}{a+2b+c} + \frac{4b}{a+b+2c} - \frac{8c}{a+b+3c}$$

6. (a) Show that among any $n+1$ whole numbers, one can find two numbers such that their difference is divisible by n .

- (b) Show that for any natural number n , there is a positive integer all of whose digits are 5 or 0 and is divisible by n .

**Good teachers
are the reason why
ordinary students
dream to do
extraordinary things...**

