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# NATIONAL LEVEL SCIENCE TALENT SEARCH EXAMINATION

Paper Code: UN 423

Solutions for Class : 12 (PCM)

## Mathematics

1. **(C)** For continuity at x = 0, we must have:

$$f(0) = \lim_{x\to 0} f(x) = \lim_{x\to 0}(1+2)^{2/x} = \lim_{x\to 0}\left\{(1+x)^{\frac{1}{x}}\right\}^2$$

$$= e^2 \qquad \left[\because \lim_{x\to 0}(1+x)^{1/x} = e\right]$$

∴ The correct answer is (c).

2. **(A)** Put x – 2 = t, so that dx = dt and ((1 – x) (x – 2) = (1 –(2+ t)) t = –t² – t

$$= \frac{1}{4} - \left(t + \frac{1}{2}\right)^2$$

$$\therefore I = \sin^{-1}\left(\frac{t+1/2}{1/2}\right) + C$$

$= \sin^{-1}(2x - 3) + C$

Alternative Solution

Put $t = \frac{1}{2}(x - 1 + x - 2) = x - \frac{3}{2}$, so that

dt = dx

$$\text{and } (1-x)(x-2) = \left(-t\frac{1}{2}\right)\left(t - \frac{1}{2}\right)$$

$$= \left(\frac{1}{2}\right)^2 - r^2$$

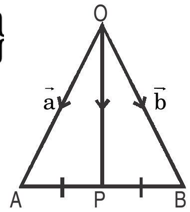
$$\therefore \qquad I = \sin^{-1}\left(\frac{t}{1/2}\right) + C$$

$$= \sin^{-1}(2x - 3) + C$$

3. **(B)** The position of the vector of the midpoint P of the line joining the points A and B having position vectors $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ is given by $\vec{p} = \frac{1}{2}(\vec{a} + \vec{b})$

$$= \frac{1}{2}\left\{\left(\hat{i} + \hat{j} - \hat{k}\right) + \left(\hat{i} - \hat{j} + \hat{k}\right)\right\}$$

$$= \frac{1}{2}(2\hat{i}) = \hat{i}$$




4. **(B)**

$$f\left(g\left(\frac{8}{5}\right)\right) - g\left(f\left(\frac{-8}{5}\right)\right) = f\left(\left|\frac{8}{5}\right|\right) - g\left(\left|\frac{-8}{5}\right|\right)$$

$$= f\left(\frac{8}{5}\right) - g(-2)$$

$$\left[\frac{-8}{5} = -1.6 \text{ and so} \left[\frac{-8}{5}\right] = -2\right]$$

$$= \left[\frac{8}{5}\right] - |-2| = 1 - 2 = -1.$$

$$\left[\frac{8}{5} = 1.6 \text{ and so} \left[\frac{8}{5}\right] = 1\right]$$

5. **(B)** We have from the given equation

$$\tan^{-1}\frac{(a+b)x}{x^2 - ab} = \frac{\pi}{2} - \tan^{-1}\frac{(c+d)x}{x^2 - cd}$$

$$\Rightarrow \tan^{-1}\frac{(a+b)x}{x^2 - ab} = \cot^{-1}\frac{(c+d)x}{x^2 - cd}$$

$$= \tan^{-1}\frac{x^2 - cd}{(c+d)x}$$

$$\Rightarrow (x^2 - ab)(x^2 - cd) = (a+b)(c+d)x^2$$

$$\Rightarrow x^4 - x^2\sum ab + abcd = 0$$

6. **(C)** $\begin{vmatrix} x & 2 & -1 \\ 2 & 5 & x \\ -1 & 2 & x \end{vmatrix} = 0$

$\Rightarrow x(5x - 2x) - 2(2x + x) - 1(4 + 5) = 0$

$\Rightarrow 3x^2 - 6x - 9 = 0$

$\Rightarrow x^2 - 2x - 3 = 0$

$\Rightarrow (x - 3)(x + 1) = 0$

$\Rightarrow x = 3$ or $x = -1$.

7. **(B)** The value of determinant of a unit matrix of any order is equal to 1.

8. **(A)** $f(x) = \frac{x}{3} + \frac{3}{x}, x \in (-3, 3)$

$\Rightarrow f'(x) = \frac{1}{3} - \frac{3}{x^2} = \frac{x^2 - 9}{3x^2}$

Clearly when $x \in (-3, 3)$, then $x^2 < 9$, i.e., $x^2 - 9 < 0$

$f'(x) < 0 \ \forall \ x \in (-3, 3)$ and so

$f(x)$ is decreasing in the interval $(-3, 3)$.

9. **(A)** The parametric form of the given equation is $x = t, y = t^2$. The equation of any tangent at t is $2xt = y + t^2$. Differentiating, we get $2t = y_1$. Putting this value in the equation of tangent, we have

$2x\,y_1/2 = y + (y_1/2)^2 \Rightarrow 4xy_1 = 4y + y_1^2$

The order of this equation is one.

10. **(D)** $A = \{x, y\}$

The subsets of A are $\phi$, $\{x\}, \{y\}, \{x, y\}$

$\therefore$ The power set of $A = \{\phi, \{x\}, \{y\}, \{x, y\}\}$

11. **(B)** We have $I_0 = \int_0^1 e^x\,dx = e^x]_0^1 = e - 1$

and for $n \geq 1$,

$I_n = e^x(x-1)^n]_0^1 - n\int_0^1 e^x(-1)^{n-1}\,dx$

$= -(-1)^n - nI_{n-1}$

$\therefore I_1 = 1 - (1)\,I_0 = 1 - (e-1) = 2 - e$

$I_2 = -1\ 2I = -1 - 2(2 - e) = 2e - 5$

and $I_3 = -1 - 3I_2 = 1 - 3(2e - 5)$

$= 16 - 3e$

$\therefore n = 3$

12. **(D)** Putting $v = y/x$ so that $x\frac{dv}{dx} + v = \frac{dy}{dx}$, we have

$$x\frac{dv}{dx} + v = v + \phi(1/v) \Rightarrow \frac{dv}{\phi(1/v)} = \frac{dx}{x}$$

$\Rightarrow \log|Cx| = \int \frac{dv}{\phi(1/v)}$ (C being constant of integration)

But $y = \frac{x}{\log|Cx|}$ is the general solution

so $\frac{x}{y} = \frac{1}{v} = \log|Cx| = \int \frac{dv}{\phi(1/v)}$

$\Rightarrow \phi(1/v) = -1/v^2$

$\Rightarrow \phi(x/y) = -y^2/x^2$

13. **(D)** The given vectors are co-planar if

$$\begin{vmatrix} 5 & 6 & 7 \\ 7 & -k & 9 \\ 3 & 20 & 5 \end{vmatrix} = 0$$

$\Rightarrow 5(-5k - 180) - 6(35 - 27) + 7(140 + 3k) = 0$

$\Rightarrow -4k + 32 \Rightarrow k = 8$

14. **(B)** $fog = 1 \Rightarrow (fog)(x) = 1(x) = x$

$\Rightarrow f\{g(x)\} = x$

$\Rightarrow f'\{g(x)\}.\ g'(x) = 1$

$\Rightarrow f'\{g(a)\}.\ g'(a) = 1$

$\Rightarrow f'(b).\ 2 = 1$

$\Rightarrow f'(b) = \frac{1}{2}$

$\therefore$ The correct answer is (b).

15. **(C)** $PQ = \begin{pmatrix} i & 0 & -i \\ 0 & -i & i \\ -i & i & 0 \end{pmatrix}\begin{pmatrix} -i & i \\ 0 & 0 \\ i & -i \end{pmatrix}$

$$= \begin{pmatrix} -i^2 + 0 - i^2 & i^2 + 0 + i^2 \\ 0 + 0 + i^2 & 0 + 0 - i^2 \\ i^2 + 0 + 0 & -i^2 + 0 + 0 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & -2 \\ -1 & 1 \\ -1 & 1 \end{pmatrix}.$$

16. **(C)** Let $S$ be the area and $l$ be the length of each side of square metal sheet.

Then, $S=l^2 \Rightarrow \frac{ds}{dt}=2l\frac{dl}{dt}$.

When $l$ = 10 cm and $\frac{dl}{dt}$ = 3 cm/s, we have:

$\left(\frac{ds}{dt}\right)=(2\times10\times3)$ sq.cm/s = 60 sq.cm/s.

17. **(A)** Put x = tan θ so that $\sqrt{x^2+1}=\sec\theta$

$dx = \sec^2\theta\, d\theta$

$$\therefore \quad I = \int\frac{\sec\theta\sec^2\theta}{\tan^4\theta}d\theta=\int\frac{\cos\theta}{\sin^4\theta}d\theta$$

$$= \frac{1}{3}\frac{1}{\sin^3\theta}+C$$

$$= \frac{1}{3}\frac{\sec^3\theta}{\tan^3\theta}+C=-\frac{1}{3}\frac{(x^2+1)^{3/2}}{x^3}+C$$

18. **(C)** A is orthogonal $\Rightarrow A.A^T = I_n$

$\therefore |A - I_n| = |A - AA^T| = |A(I_n - A^T)| = |A|\,|I_n - A^T|$

$= |A|\,|I_n - A|$

[$\because$ $|I_n - A| = |(I_n - A)^T| = |(I_n)^T - A^T| = |I_n - A^T|$]

19. **(B)** Putting y/x = u, we have $\frac{dy}{dx}=u+x\frac{du}{dx}$.

The given differential equation can be written as

$$u + x\frac{du}{dx}=u+\frac{\phi(u)}{\phi'(u)}\Rightarrow x\frac{du}{dx}=\frac{\phi(u)}{\phi'(u)}$$

$$\Rightarrow\frac{\phi'(u)}{\phi(u)}du=\frac{dx}{x}.$$

Intergrating, we get log $\phi(u)$ = log x + log k, so $\phi(u)$ = k x, i.e., $\phi(y/x)$ = kx.

20. **(A)** R is Reflexive since $(x, x)\in R\ \forall\ x\in A$.

R is not symmetric since $(1, 2)\in R$ but $(2, 1)\notin R$.

R is not Transitive since $(3, 1)\in R, (1, 2)\in R$ but $(3, 2)\notin R$.

21. **(D)** f (x) = a sin x + b cos x

f ' (x) = a cos x – b sin x

f '' (x) = –a sin x – b cos x

Now, f ' (x) = 0 $\Rightarrow$ a cos x – b sin x = 0 $\Rightarrow$ a cos x = b sin x

$$\Rightarrow \tan x = \frac{a}{b}$$

$$\Rightarrow\left(\sin x=\frac{a}{\sqrt{a^2+b^2}},\cos x=\frac{b}{\sqrt{a^2+b^2}}\right)$$

$$\text{or}\ \left(\sin x=\frac{-a}{\sqrt{a^2+b^2}},\cos x=\frac{-b}{\sqrt{a^2+b^2}}\right)$$

$$\left[\tan x=\frac{a}{b}\Rightarrow\sin x=\pm\frac{a}{\sqrt{a^2+b^2}},\cos x=\pm\frac{b}{\sqrt{a^2+b^2}}\right]$$

When $\sin x=\frac{a}{\sqrt{a^2+b^2}},\cos x=\frac{b}{\sqrt{a^2+b^2}}$,

we have :

f '' (x) =

$$-a\left(\frac{a}{\sqrt{a^2+b^2}}\right)-b.\left(\frac{b}{\sqrt{a^2+b^2}}.\right)=-\sqrt{a^2+b^2}<0$$

$\therefore$ This is a point of maximum.

When $\sin x=\pm\frac{-a}{\sqrt{a^2+b^2}},\cos x=\pm\frac{-b}{\sqrt{a^2+b^2}}$,

we have :

f '' (x) =

$$-a\left(\frac{-a}{\sqrt{a^2+b^2}}\right)-b.\left(\frac{-b}{\sqrt{a^2+b^2}}.\right)=-\sqrt{a^2+b^2}>0.$$

$\therefore$ This is a point of maximum.

Maximum value of

$$f(x) = a.\left(\frac{a}{\sqrt{a^2+b^2}}\right)+b.\left(\frac{b}{\sqrt{a^2+b^2}}\right)=\sqrt{a^2+b^2}$$

Note: we get the minimum value of f (x) when

$$\sin x=\frac{-a}{\sqrt{a^2+b^2}},\cos x=\frac{-b}{\sqrt{a^2+b^2}}$$

Minimum value of

$$f(x)=a.\left(\frac{-a}{\sqrt{a^2+b^2}}\right)+b.\left(\frac{-b}{\sqrt{a^2+b^2}}\right)=-\left(\sqrt{a^2+b^2}\right)$$

Thus, for a function f (x) = a sin x + b cos x, the minimum value is $-(\sqrt{a^2+b^2})$ and the maximum value is $(\sqrt{a^2+b^2})$.

22. **(C)** $|\vec{a} \times \vec{b}| = 0$

$\Rightarrow |\vec{a}|\,|\vec{b}| \sin\theta$ where $\theta$ is the angle between $\vec{a}$ and $\vec{b}$

$\Rightarrow |\vec{a}| = 0$ or $|\vec{b}| = 0$ or $\sin\theta = 0$

Also, $\vec{a}.\vec{b} = 0$

$\Rightarrow |\vec{a}|\,|\vec{b}| \cos\theta = 0$ where $\theta$ is the angle between $\vec{a}$ and $\vec{b}$

$\Rightarrow |\vec{a}| = 0$ or $|\vec{b}| = 0$ or $\cos\theta = 0$

Since $\sin\theta = 0$ and $\cos\theta = 0$ are simultaneously not possible for value of $\theta$, so we have :

$|a \times b| = a.b \Rightarrow$ either $|a| = 0$ or $|b| = 0$

23. **(B)** For continuity at x =1, we must have

$f(1) = f(1+0) = f(1-0)$.

$f(1) = (a \times 1^2 + b) = (a + b)$

$f(1+0) = \lim_{h\to 0} f(1+h)$

$= \lim_{h\to 0} \{b(1+h)^2 + a(1+h) + c\}$

$= \lim_{h\to 0} \{bh^2 + (2b+a)h + (b+a+c)\}$

$= b + a + c$

$\therefore f(1) = f(1+0) \Rightarrow (a+b) = b + a + c$

$\Rightarrow c = 0$.

Now, f(x) being differentiable at x =1, we have $Rf'(1) = Lf'(1)$,

$$Rf'(1) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$$

$$= \lim_{h\to 0} \frac{b(1+h)^2 + a(1+h) + c - (a+b)}{h}$$

$$= \lim_{h\to 0} \frac{bh^2 + (2b+a)h + c}{h}$$

$$\left[\text{Form } \frac{0}{0}, \text{otherwise } \lim = \infty\right]$$

$$= \lim_{h\to 0} \frac{2bh + (2b+a)}{1} = (2b+a)$$

$$Lf'(1) = \lim_{h\to 0} \frac{f(1-h)-f(1)}{-h}$$

$$= \lim_{h\to 0} \frac{a(1-h)^2 + b - (a+b)}{-h}$$

$$= \lim_{h\to 0} \frac{ah^2 - 2ah}{-h} \quad \left[\text{Form } \frac{0}{0}\right]$$

$$= \lim_{h\to 0} (-ah + 2a) = 2a.$$

$\therefore Rf'(1) = Lf'(1) \Rightarrow 2b + a = 2a \Rightarrow a = 2b$

Hence, a = 2b, c = 0

$\therefore$ The correct answer is (B)

24. **(C)** Let A be of the order m × n.

Since A + B is defined, so B is of the same order as A. i.e., B is of the order m × n.

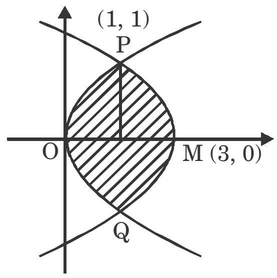
Also, since AB is defined, so the number of columns of A = number of rows of B i.e., n = m.

$\therefore$ A is of order m × m and B is of order m × m.

Hence A, B are square matrices of the same order.

25. **(D)** The two curves represent parabolas with vertices at (0, 0) and (3, 0). They intersect at (1, 1) and (1, –1), so that requried area is

area of *OPMQO* = 2 (area of *OPMO*).




$$= 2\left(\int_0^1 \sqrt{x}\,dx + \int_1^3 \sqrt{\frac{3-x}{2}}\,dx\right)$$

$$= 2\left(\frac{2}{3}x^{3/2}\Big|_0^1 - \frac{1}{\sqrt{2}}.\frac{2}{3}(3-x)^{3/2}\Big|_1^3\right)$$

$$= 2\left[\frac{2}{3} - \left(0 - \frac{1}{\sqrt{2}}.\frac{2}{3}2^{3/2}\right)\right] = 2\left(\frac{2}{3} + \frac{4}{3}\right) = 4$$

26. **(D)** If we denote $\cos^{-1}x$ by y, then

since $0 \le \cos^{-1}x \le \pi \Rightarrow 0 \le 2y \le 2\pi$ (1)

Also since

$-\pi/2 \le \sin^{-1}(2x\sqrt{1-x^2}) \le \pi/2$

$\Rightarrow -\pi/2 \le \left(\sin^{-1}\sin(2y) \le \pi/2\right)$

$\Rightarrow -\pi/2 \le 2y \le \pi/2$

(2)

From (1) and (2) we find $0 \le 2y \le \pi/2$

$\Rightarrow 0 \le y \le \pi/4 \Rightarrow 0 \le \cos^{-1}x \le \pi/4$

which holds if $1/\sqrt{2} \le x \le 1$.

27. **(C)** $R = a\sqrt{2}$

$A = a^2$

$= \left(\frac{R}{\sqrt{2}}\right)^2$

$A = \frac{R^2}{2}$

$\frac{dA}{dl} = \frac{\cancel{2}R}{\cancel{2}}$

28. **(C)** $f(x) = 4x + \frac{9\pi^2}{x} + \sin x$

$f'(x) = 4 - \frac{9\pi^2}{x^2} + \cos x$

$f''(x) = \frac{18\pi^2}{x^3} - \sin x$

Now, $f'(x) = 0 \Rightarrow 4 - \frac{9\pi^2}{x^2} + \cos x = 0$

$\Rightarrow \quad x = \frac{3\pi}{2}$

At $x = \frac{3\pi}{2}$, we have: $f''(x) =$

$f''\left(\frac{3\pi}{2}\right) = \left(\frac{16}{3\pi} + 1\right) > 0$.

$\therefore\ x = \frac{3\pi}{2}$ is a point of minima.

Least (minimum) value of f (x) =

$f\left(\frac{3\pi}{2}\right) = 6\pi + 6\pi - 1 = 12\pi - 1$

29. **(A)** Let R be a relation defined on a set A containing 6 elements.

Let A = {a, b, c, d, e, f}

Then, R must contain atleast 6 elements (ordered pairs) to be an equivalence relation. i.e., we must have

R = {(a, a), (b, b), (c, c), (d, d), (e, e), (f, f)}

30. **(B)** Put $x^2 = t$

or $x = \sqrt{t}$ or $dx\ \frac{1}{2\sqrt{t}} dt$

$\therefore I = \int_0^{2.89} \frac{[t]}{2\sqrt{t}} dt$

$= \int_0^1 \frac{[t]}{2\sqrt{t}} dt + \int_1^2 \frac{[t]}{2\sqrt{t}} dt + \int_2^{2.89} \frac{[t]}{2\sqrt{t}} dt$

$= 0 + \int_1^2 \frac{[t]}{2\sqrt{t}} dt + \int_2^{2.89} \frac{[t]}{2\sqrt{t}} dt$

$= \sqrt{t}\Big]_1^2 + 2\sqrt{t}\Big]_2^{2.89}$

$= (\sqrt{2} - 1) + 2(1.7 - \sqrt{2})$

$= 2.4 - \sqrt{2}$

31. **(D)** $\operatorname{sgn} x = \begin{cases} \frac{|x|}{x}, x \neq 0 \\ 0, x = 0 \end{cases}$

$= \begin{cases} 1, x > 0 \\ 0, x = 0 \\ -1, x < 0 \end{cases}$

$\therefore x^3 \operatorname{sgn} x = \begin{cases} x^3, x > 0 \\ 0, x = 0 \\ -x^3, x < 0 \end{cases}$

The doubtful point about continuity and differentiability is x =0

f (0) = 0,

$f(0 + 0) = \lim_{h \to 0} f(0 + h) = \lim_{h \to 0} h^3 = 0$

$f(0 - 0) = \lim_{h \to 0} f(0 - h) = \lim_{h \to 0} -(-h^3)$

$= \lim_{h \to 0} h^3 = 0$

$\therefore$ f(x) is continuous at x = 0 also.

Now $Rf'(0) = \lim_{h \to 0} \frac{f(0 + h) - f(0)}{h}$

$= \lim_{h \to 0} \frac{h^3 - 0}{h} = \lim_{h \to 0} h^2 - 0$

$Lf'(0) = \lim_{h \to 0} \frac{f(0 - h) - f(0)}{-h}$

$= \lim_{h \to 0} \frac{-(-h)^3 - 0}{-h} = \lim_{h \to 0} h^2 = 0$

$\therefore$ Rf' (0) = Lf' (0) = 0.

Hence, f(x) is derivable at x = 0.

$\therefore$ The correct answer is (D).

32. **(A)** $I = \int \frac{\sqrt{1 + \sin x}\sqrt{1 - \sin x}}{\sqrt{1 - \sin x}} dx$

$= \int \frac{\cos x}{\sqrt{1 - \sin x}} dx = -2\sqrt{1 - \sin x} + C$

33. **(D)** $dy/dx = e^{3x+4y} = e^{3x}e^{4y} \Rightarrow e^{-4y}\,dy = e^{3x} = dx$

$$\Rightarrow \frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} + C$$

Putting x = 0, we have –1/4 –1/3 = C

$\Rightarrow$ C = – 7/12

Hence $\dfrac{e^{-4y}}{-4} = \dfrac{e^{3x}}{3} - \dfrac{7}{12} \Rightarrow 7 = 3e^{-4y} + 4e^{3x}$

34. **(B)** f is one-one since

$f(x_1) = f(x_2) \Rightarrow \dfrac{x_1}{1+x_1} = \dfrac{x_2}{1+x_2} \Rightarrow x_1 + x_1x_2 = x_2 + x_1x_2 \Rightarrow x_1 = x_2$.

Let y = f(x)

$\Rightarrow y = \dfrac{x}{1+x} \Rightarrow y + xy = x \Rightarrow y = x(1-y) \Rightarrow x = \dfrac{y}{1-y}$.

Clearly, x is not defined when y = 1.

$\therefore 1 \in (0,\infty)$ has no pre-image in $(0,\infty)$.

Hence, f is not onto.

35. **(B)** $\begin{pmatrix} a & b & a-b \\ b & c & b-c \\ 2 & 1 & 0 \end{pmatrix} = \begin{pmatrix} a & b & 0 \\ b & c & 0 \\ 2 & 1 & -1 \end{pmatrix}$

$[C_3 \to C_3 - C_1 + C_2]$

$= -(ac - b^2) = b^2 - ac$.

Now, $\Delta = 0$

$\Rightarrow b^2 - ac = 0$

$\Rightarrow b^2 = ac$

$\Rightarrow$ a, b, c are in G.P.

36. **(C)** $-\dfrac{\pi}{2} \le \sin^{-1} x \le \dfrac{\pi}{2}$

$\Rightarrow -\pi/4 \le \sin^{-1} a \le \pi/4$

$\Rightarrow -(1\sqrt{2}) \le a \le (1/\sqrt{2})$

37. **(C)** $s = 3r^2 - 8r + 5$

$\Rightarrow$ velocity $v = \dfrac{ds}{dt} = 6t - 8$

Now, the body will stop when velocity becoems zero.

$v = 0 \Rightarrow 6t - 8 = 0 \Rightarrow t = \dfrac{8}{6}\text{sec} = \dfrac{4}{3}\text{sec}$.

38. **(B)** $f(x) = \dfrac{1-x}{1+x} \Rightarrow y = \dfrac{1-x}{1+x}$ where y = f(x)

$\Rightarrow y + xy = 1 - x \Rightarrow x(1+y) = 1 - y$

$\Rightarrow x = \dfrac{1-y}{1+y}$

$\Rightarrow f^{-1}(y) = \dfrac{1-y}{1+y}$ $[\because y = f(x) \Rightarrow x = f^{-1}(y)]$

$\Rightarrow f^{-1}(y) = \dfrac{1-x}{1+x} = f(x)$

39. **(D)** We can write

$$I = \int_0^a \frac{a-x}{\sqrt{a^2-x^2}}\,dx$$

$$= \left[a\sin^{-1}\left(\frac{x}{a}\right) + \sqrt{a^2-x^2}\right]_0^a$$

$$= a\left(\frac{\pi}{2} - 1\right)$$

40. **(C)** Given that

$(a+b)^m = {}^mC_0A^m + {}^mC_1A^{m-1}B + {}^mC_2A^{m-2}B^2 + \dots + {}^mC_mB^m$ .... (i)

Putting m = 2 in (i) we get:

$(A + B)^2 = {}^2C_0A^2 + {}^2C_1A^B + {}^2C_2B^2 = A^2 + 2AB + B^2$ .... (ii)

But we have:

$(A + B)^2 = (A + B).(A + B) = A^2 + AB + BA + B^2$ ....... (iii)

From (ii) and (iii) we get:

$2AB = AB + BA \Rightarrow AB = BA$.

41. **(C)** **In air:** $\beta$ = 2.0 mm = $2.0 \times 10^{-3}$ m;

$\lambda$ = 6000 Å = $6 \times 10^{-7}$ m;

Now, $\beta = \dfrac{D\lambda}{d}$

or $\dfrac{D}{d} = \dfrac{\beta}{\lambda} = \dfrac{2.0\times10^{-3}}{6\times10^{-7}}$

**In liquid:** Let $\lambda'$ be the wavelength of light in liquid and $\beta'$ be the corresponding fringe width. Then,

$\lambda' = \dfrac{\lambda}{\mu} = \dfrac{6\times10^{-7}}{1.33}$

Now,

$\beta' = \dfrac{D\lambda'}{d} = \dfrac{2.0\times10^{-3}}{6\times10^{-7}} \times \dfrac{6\times10^{-7}}{1.33}$

= $1.5 \times 10^{-3}$ m

= 1.5 mm.

42. **(C)** Frequency of emitted radiation $v \propto Z^2$

$\therefore \dfrac{V_H}{V_{Li}} = \left(\dfrac{Z_H}{Z_{Li}}\right)^2$

$= \left(\dfrac{1}{3}\right)^2$

$\therefore v_{Li} = 9v_H$

$= 9v_0$

43. **(A)** $v = L\dfrac{di}{dt}$ $\Rightarrow 4t = L\dfrac{di}{dt}$

$\dfrac{1}{L}\int_0^4 4t\,dt = \int_0^i di$ $\Rightarrow i = 32A$

Energy stored, U = $\dfrac{1}{2}Li^2 = \dfrac{1}{2} \times 1 \times 32^2$

= 512 J

44. **(C)** Maximum magnetic field is at the surface and is given by

$B = \dfrac{\mu_0}{2\pi}\dfrac{i}{R}$, $\therefore i = \dfrac{BR}{(\mu_0/2\pi)}$

$= \dfrac{(5\times10^{-3})\left(\dfrac{1.6}{2}\times10^{-3}\right)}{2\times10^{-7}}$

= 20 A

45. **(A)** Dipole moment = $p = 2l\,q = 1 \times 10^{-2} \times 2 \times 10^{-6} = 2 \times 10^{-8}$ cm.

Electric intensity, E = $10^3$ NC$^{-1}$

$\theta = 30°$

Torque, $\tau = pE \sin\theta = 2 \times 10^{-8} \times 10^3 \sin 30°$

$= 2 \times 10^{-8} \times 10^3 \times \dfrac{1}{2} = 10^{-5}$ Nm

46. **(B)** Here, E = 10 V; r = 3 Ω; I = 0.5 A

Let R be the resistance of the resistor. Then,

$I = \dfrac{E}{R+r}$

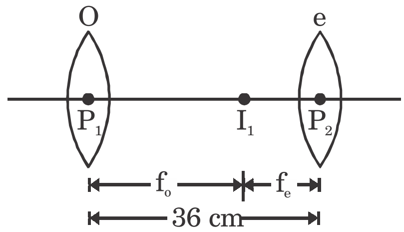
or $R = \dfrac{E}{I} - r - \dfrac{10}{0.5} - 3 = 17\Omega$

Now, the terminal voltage of the battery

= potential difference across R

= IR = 0.5 × 17 = 8.5 V

47. **(D)** Image formed by objective ($I_1$) is at second focus of it because objective is focussed for distant objects. Therefore,




Further $I_1$ should lie at first focus of eyepiece because final image is formed at infinity.

$\therefore P_2I_1 = f_e$

Given $P_1P_2$ = 36 cm

$\therefore f_0 + f_e = 36$ ............ (1)

Further angular magnification is given as 5. Therefore,

$\dfrac{f_0}{f_e} = 5$ ............ (2)

Solving Eqs. (1) and (2), we get

$f_0$ = 30 cm and $f_e$ = 6 cm.

48. **(A)** The energy of a photon of wavelength 500 nm is

$\dfrac{hc}{\lambda} = \dfrac{1242 \text{ eV} - \text{nm}}{500 \text{ nm}} = 2.48$ eV

The energy of a photon of wavelength 700 nm is

$\dfrac{hc}{\lambda} = \dfrac{1242 \text{ eV} - \text{nm}}{700 \text{ nm}} = 1.77$ eV

The energy difference between the states involved in the transition should, therefore, be between 1.77 eV and 2.48 eV.

n = 4, E = – 0.85 eV
n = 3, E = – 1.5 eV
n = 2, E = – 3.4 eV
n = 1, E = – 13.6 eV

The given figure shows some of the energies of hydrogen states. It is clear that only those transitions which end at n = 2 may emit photons of energy between 1.77 eV and 2.48 eV. Out of these only n = 3 $\rightarrow$ n = 2 falls in the proper range. The energy of the photon emitted in the transition n = 3 to n = 2 is $\Delta E$ = (3.4 – 1.5) eV = 1.9 eV. The wavelength is

$$\lambda = \frac{hc}{\Delta E}$$

$$= \frac{1242 \text{ eV} - \text{nm}}{1.9 \text{eV}} = 654 \text{ nm}.$$

49. **(C)** Here, R = 30 ohm; $X_L$ = 40 ohm

Therefore, impedance of the LR-circuit,

$$Z = \sqrt{R^2 + X_L{}^2} = \sqrt{(30)^2 + (40)^2}$$

$$= \sqrt{900 + 1600}$$

$$= \sqrt{2500} = 50 \text{ ohm}$$

Power factor, $\cos\phi = \frac{R}{Z} = \frac{30}{50} = 0.6$

Now, $E_0$ = 220 volt; $I_0$ = 1 ampere

$\therefore P_{av} = E_v I_v \cos\phi$

$$= \frac{E_0}{\sqrt{2}} \times \frac{I_0}{\sqrt{2}} \cos\phi = \frac{220 \times 1 \times 0.6}{\sqrt{2} \times \sqrt{2}} = 66 \text{ watt}$$

50. **(D)** The energy density is

$u_{av} = \frac{1}{2}\varepsilon_0 E_0^2 = \frac{1}{2} \times (8.85 \times 10^{-12}\ C^2/N\text{–}m^2) \times (50\ N/C)^2$

$= 1.1 \times 10^{-8}\ J/m^3$

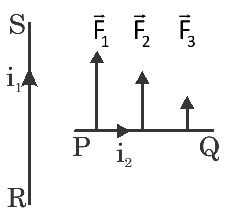
The volume of the cylinder is V = 10 cm$^2$ × 50 cm = 5 × 10$^{-4}$ m$^3$.

The energy contained in this volume is

$U = (1.1 \times 10^{-8}\ J/m^3) \times (5 \times 10^{-4} m^3)$

$= 5.5 \times 10^{-12}$ J.

51. **(C)** Every current carrying loop is a magnetic dipole. If it lies in the plane of paper and current in it is in clockwise direction magnetic field at all points lying within the loop is perpendicular to paper inwards and at points outside the loop magnetic field is perpendicular to paper in outward direction. For $\theta$ < 180°, the centre O lies outside the loop and current is clockwise. Therefore, magnetic field is perpendicular to paper in outward direction.




52. **(D)** Let the potential be zero at the point *P*, at a distance *x* from the charge $q_1$.

$q_1 = -4\mu C = -4 \times 10^{-6}\ C,\ q_2 = 2 \times 10^{-6}\ C$

$Ap = x,\ PB = (1 - x)m$

Potential due to $q_1$ at P is $V_A = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1}{x}$

Potential due to $q_2$ at *P* is

$$V_B = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_2}{(1-x)}$$

Total potential at *P* is zero

$$\frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1}{x} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_2}{(1-x)} = 0$$

$$\frac{q_1}{x} = -\frac{q_2}{(1-x)}$$

$$\frac{-4 \times 10^{-6}}{x} = \frac{-2 \times 10^{-6}}{1-x}$$

$2(1 - x) = x$

$2 - 2x = x$

$3x = 2$

$x = \frac{2}{3} = 0.667$ m

The electric potential is zero at a point distance 0.667 m from the – 4 μC charge.

53. **(B)** Amount left after 10 years

$$A = A_0 \left(\frac{1}{2}\right)^{t/T_{1/2}}$$

$$= A_0 \left(\frac{1}{2}\right)^{10/5}$$

$$= A_0 \left(\frac{1}{2}\right)^2 = \frac{A_0}{4}$$

Probability of decay $= \frac{A_0 - A_0/4}{A_0} \times 100\%$

$= \frac{3}{4} \times 100 = 75\%$

54. **(A)** When current flows in any of the coils, the flux linked with the other coil will be maximum in the first case. Therefore, mutual inductance will be maximum in case (a).

55. **(A)** For real image

$u = -u_1,\ v = -2u_1,\ f = -20$ cm

Substituting in $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

We get $\frac{1}{-2u_1} - \frac{1}{u_1} = -\frac{1}{20}$

or $u_1$ = 30 cm

For vitual image $u = -u_2$, $v = 2u_2$, $f = -20$ cm

$$\therefore \frac{1}{2u_2} - \frac{1}{u_2} = -\frac{1}{20}$$

or $u_2 = 10$ cm

$\therefore$ Distance between two positions of the object are $u_1 - u_2$

or 30 cm – 10 cm = 20 cm.

56. **(A)** The energy of the photon is $E = \frac{hc}{\lambda}$

$$= \frac{1242 \text{ eV} - \text{nm}}{589 \text{ nm}} = 2.1 \text{ eV}.$$

Hence, the minimum energy required to create a hole-electron pair is 2.1 eV.

57. **(D)** Total magnetic flux passing through whole of the X-Y plane will be zero, because magnetic lines form a closed loop. So as many lines will move in –Z direction and same will return to +Z direction from the X-Y plane.

58. **(B)** Radius of the loop = $r = \frac{20}{2}$ = 10 cm

= 0.10 m

Total flux = $\phi$ = $1.3 \times 10^5$ Nm²/C

But $\phi = E.\pi r^2$

$$E = \frac{\phi}{\pi r^2} = \frac{1.3 \times 10^5}{3.14 \times (0.10)^2} = 4.14 \times 10^6 \text{ N/C}$$

59. **(B)** Here, work function,

$\omega$ = 1.2 eV = $1.2 \times 1.6 \times 10^{-19} = 1.92 \times 10^{-19}$ J

Wavelength of the incident radiation,

$\lambda$ = 5000 Å = $5000 \times 10^{-10}$ m

If photoelectrons are emitted with maximum velocity $v_{max}$, then

$$\frac{1}{2}mv_{max}^2 = hv - \omega = \frac{hc}{\lambda} - \omega$$

$$= \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{5000 \times 10^{-10}} - 1.92 \times 10^{-19}$$

$= 3.97 \times 10^{-19} - 1.92 \times 10^{-19} = 2.05 \times 10^{-19}$ J

If e is the charge on electron and $V_0$ is the stopping potential, then $eV_0 = \frac{1}{2}mv_{max}^2 = 2.05 \times 10^{-19}$ J

$$\text{or } V_0 = \frac{2.05 \times 10^{-19}}{e}$$

$$= \frac{2.05 \times 10^{-19}}{1.6 \times 10^{-19}} = 1.28 \text{ V}$$

60. **(C)** Here, $I_v$ = 1.5 mA = $1.5 \times 10^{-3}$ A, $\omega$ = 300 rad s⁻¹,

R = 10 kΩ = $10^4$ Ω and C = 0.5 μF = $0.5 \times 10^{-6}$ F

Now,

$$X_C = \text{know } \frac{1}{\omega C} = \frac{1}{300 \times 0.5 \times 10^{-6}}$$

$$= 6.667 \times 10^3 \Omega$$

Therefore, r.m.s. voltage across the capacitor,

$V_C = I_v \times X_C = 1.5 \times 10^{-3} \times 6.667 \times 10^3 = 10$ V

$$\text{Also, } Z = \sqrt{R^2 + X_C^2} = \sqrt{(10^4)^2 + (6.667 \times 10^3)^2}$$

$$= 1.2 \times 10^4 \Omega$$

61. **(C)** For a given length and area of cross-section, the resistance of a material is directly proportional to its specific resistance. Since, the specific resistance is least for silver, it is the best conductor of the three given materials.

62. **(C)** In any nuclear reaction mass number and atomic number should remain conserved. Reaction (C) satisfies this condition. Also for $^{239}_{93}Np$, neutron to proton ratio is greater than 1.52 which makes it unstable.

63. **(A)** $l$ = 1 m, m = 0.2 Kg, I = 10 A

(a) The tension in the wire is zero when the magnetic force on the wire $BIl$ is equal and opposite to its weight mg.

$$B = \frac{mg}{Il} = \frac{0.2 \times 9.8}{10 \times 1} = 0.196 \text{ T}$$

(b) When the current is reversed, the magnetic force $BIl$ and mg act vertically downwards. The tension in the wire.

$= T = BIl + mg = 2mg = 2 \times 0.2 \times 9.8$

= 3.92 N.

64. **(D)** $\lambda$ = 6000 Å = $6000 \times 10^{-10}$ m, D = 1 m, $\beta$ = 0.5 mm = $0.5 \times 10^{-3}$ m

$$d = \frac{\lambda D}{\beta} = \frac{6000 \times 10^{-10}}{0.5 \times 10^{-3}} = 1.2 \times 10^{-3} \text{ m}.$$

65. **(C)** $C_1 = 3\ \mu F =, C_2 = 5\ \mu F, C_3 = 7\ \mu F$

(i) The effective capacitance when connected in series = $C_s$ = ?

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{3} + \frac{1}{5} + \frac{1}{7} = 0.676$$

$$C_s = \frac{1}{0.676} = 1.48\ \mu F$$

(ii) The effective capacitance when connected in parallel = $C_p$ = ?

$C_p = C_1 + C_2 + C_3 = 3 + 5 + 7 = 15\ \mu F$

## Chemistry

66. **(A)** $Al^{3+} + 3e^- \rightarrow Al.$

One mole requires 3 × 96500 C

$\therefore$ One millimole requires = 3 × 96.5 C

= 9.65 A × time in sec

$$\therefore \text{Time} = \frac{3 \times 96.5}{9.65} = 30\ s$$

67. **(A)** The decreasing order of the boiling points of given hydrides is

$SbH_3 > NH_3 > AsH_3 > PH_3$

68. **(A)** The coordination number of the central atom or ion is determined by the number of sigma bonds between the ligands and the central atom/ion.

69. **(A)**

$$\underset{\text{Ethyl isocyanide}}{C_2H_5N \rightleftarrows C} + H_2O \xrightarrow{H} \underset{\text{Ethylamine}}{C_2H_5NH_2} + \underset{\text{Methanoic acid}}{HCOOH}$$

70. **(B)** $CH_3CH_2CH_2COOH > CH_3CH_2CH_2CH_2OH > CH_3CH_2CH_2CHO$

(a) The – OH group of the – COOH group in an acid is more strongly polarized due to the presence of carbonyl oxygen adjacent to it.

(b) Alcohol molecules can form H-bonds through their –OH groups. The presence of carbonyl group in acids permits the formation of H-bonds through –OH and the carbonyl oxygen. Thus, a carboxylic acid molecule can enter into hydrogen bonding at two different points.

71. **(D)** The helical structure of proteins is stabilized by NH ---- C=O hydrogen bonds.

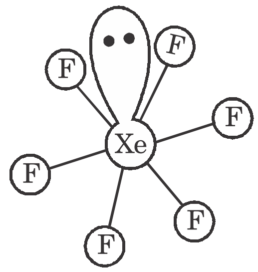
72. **(A)** Amount of NaCl = $\dfrac{1.00\ g}{58.5\ g / mol}$

No. of unit cells in 1.00 g of NaCl

$$= \frac{6.02 \times 10^{23} mol^{-1}}{4} \times \frac{1.00\ g}{58.5\ g\ mol^{-1}}$$

$= 2.57 \times 10^{21}$

73. **(A)** $XeF_6$ has seven electron pairs (6 bonding pairs and one lone pair) and would, thus, have a distorted octahedral structure as found experimentally in the gas phase.




74. **(C)** Due to stabilization of –COOH by resonance, electrophilicity of C-atom of C = O group of COOH decreases and hence carboxylic acids do not give characteristic reactions (such as cyanohydrin formation, reaction with $NH_2OH$, etc) of aldehydes/ ketones.

75. **(A)** According to Henry's law for a gas at 2 different concentrations

$$k = \frac{C_1}{P_1}, \quad k = \frac{C_2}{P_2}$$

$$\frac{C_1}{P_1} = \frac{C_2}{P_2}$$

Given that $C_1$ = 0.015 g/L

$P_1$ = 580 torr, $P_2$ = 800 torr, $C_2$ = ?

$$\text{So, } \frac{0.015}{580} = \frac{C_2}{800}$$

$C_2$ = 0.0207 g/L

76. **(A)** RCl and RBr can be prepared by free radical halogenation of alkanes while RF and RI cannot be prepared. With $F_2$, the reaction is not only explosive but also brings cleavage of C-C bonds while with $I_2$ the reaction is too slow to be of any practical value.

77. **(D)** In a bessemer converter, copper pyrites are oxidised to FeO and $Cu_2O$. FeO is slagged off. $Cu_2O$ reacts with $Cu_2S$ left unoxidised to give Cu.

$2\ Cu_2S + 3\ O_2 \rightarrow 2\ Cu_2O + 2\ SO_2 \uparrow$

$2\ Cu_2O + Cu_2S \rightarrow 6\ Cu + SO_2 \uparrow$

78. **(C)** Zinc ($3\ d^{10}$) configuration shows only +2 ox. state.

79. **(D)** Ketones are less reactive because of statements (B) and (C).

80. **(B)** Argentite. $Ag_2S$ is concentrated by leaching with NACN solution in the presence of air.

$2\ Ag_2S + 8\ NaCN + O_2 + 2H_2O \rightarrow$

$4\ Na[Ag(CN)_2] + 4\ NaOH + 2S$

81. **(C)** Both optical and geometrical isomerisms are the types of streoisomerism.

82. **(C)** Rate = k $[NO]^2 [O_2]$

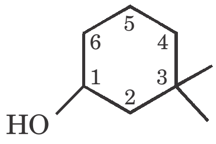
New rate = k $(2[NO])^2 (2[O_2])$

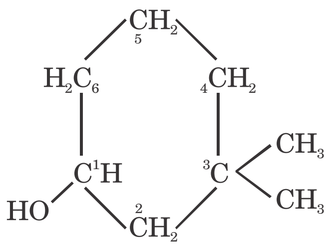
$= k [NO]^2 [O_2].8$

New rate = 8 × Rate

83. **(D)** Low temperature and high pressure favours the conversion of $SO_2$ to $SO_3$ in contact process for the manufacture of $H_2SO_4$.

84. **(C)** The IUPAC name of the given compound is 3, 3-dimethyl-1-cyclohexanol.




85. **(C)**

$$C_6H_5NH_2 + CH_3COCl \longrightarrow \underset{\text{Acetanilide}}{C_6H_5NHCOCH_3} + HCl.$$

86. **(A)** The two structures are non-superimposable mirror images of each other and hence are enantiomers.

87. **(B)** As equal number of $Na^+$ and $Cl^-$ ions are missing from their lattice site, it is Schottky defect.

88. **(C)** Since, compound A undergoes Cannizzaro reaction, it must be formaldehyde. Since, compound B undergoes iodoform test, it must be a methyl ketone, i.e., 2-pentanone.

89. **(B)** Adsorption is an exothermic process. So, it decreases with an increase in temperature.

90. **(C)** In HF, the molecules are associated through hydrogen bonding resulting in increase of b.p. of the liquid.