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NATIONAL LEVEL SCIENCE TALENT SEARCH EXAMINATION

Paper Code: UN 423

Solutions for Class : 11 (PCM)

Mathematics

1. (B) The given equation is equivalent to
 $2 \sin^2 ((\pi/2) \cos^2 x) = 2 \sin^2 ((\pi/2) \sin 2x)$
 $\Rightarrow \cos^2 x = \sin 2x$
 $\Rightarrow \cos x (\cos x - 2 \sin x) = 0$
 $\Rightarrow 1 - 2 \tan x = 0$ as $\cos x \neq 0$,
 $x \neq (2n+1) \pi/2$
 $\Rightarrow \tan x = 1/2$
 $\Rightarrow \cos 2x = \frac{1 - \tan^2 x}{1 + \tan^2 x} = \frac{3}{5}$
2. (D) The index of $(x - a)^8$ is 8 which are even and so the middle term in the expansion of $(x - a)^8$ is $\left(\frac{8}{2} + 1\right)^{\text{th}}$ term i.e., 5th term.
 Now, the general term in the expansion of $(x - a)^8$ is
 $t_{r+1} = {}^8C_r (-1)^r x^{8-r} a^r$
 Putting $r = 4$ in (i), we get
 $t_5 = {}^8C_4 (-1)^4 x^4 a^4 = {}^8C_4 x^4 a^4$.
3. (A) $\log_{\sqrt{3}} x + \log_{\sqrt[4]{3}} x + \log_{\sqrt[6]{3}} x + \dots + \log_{\sqrt[16]{3}} x = 36$
 $\Rightarrow \frac{\log x}{\log \sqrt{3}} + \frac{\log x}{\log \sqrt[4]{3}} + \frac{\log x}{\log \sqrt[6]{3}} + \dots + \frac{\log x}{\log \sqrt[16]{3}} = 36$
 $\Rightarrow \frac{\log x}{\log 3^{1/2}} + \frac{\log x}{\log 3^{1/4}} + \frac{\log x}{\log 3^{1/6}} + \dots + \frac{\log x}{\log 3^{1/16}} = 36$
 $\Rightarrow \frac{\log x}{\frac{1}{2} \log 3} + \frac{\log x}{\frac{1}{4} \log 3} + \frac{\log x}{\frac{1}{6} \log 3} + \dots + \frac{\log x}{\frac{1}{16} \log 3} = 36$
 $\Rightarrow \left(\frac{\log x}{\log 3}\right) \{2 + 4 + 6 + \dots + 16\} = 36$
 $\Rightarrow \left(\frac{\log x}{\log 3}\right) \left\{\frac{8}{2}(2 + 16)\right\} = 36$
 $\Rightarrow \log x = \frac{1}{2} \log 3 \Rightarrow x = \sqrt{3}$.

4. (B) Given limit

$$= \lim_{x \rightarrow 0} \frac{x^{1/2} \left(x + \frac{x^3}{3} + \frac{2}{15} x^5 + \dots \right)}{\left\{ \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \right) - 1 \right\}^{3/2}}$$

$$= \lim_{x \rightarrow 0} \frac{x^{3/2} \left(1 + \frac{x^2}{3} + \frac{2}{15} x^4 + \dots \right)}{x^{3/2} \left(1 + \frac{x}{2} + \frac{x^2}{6} + \dots \right)^{3/2}}$$

$$= \lim_{x \rightarrow 0} \frac{\left(1 + \frac{x^2}{3} + \frac{2}{15} x^4 + \dots \right)}{\left(1 + \frac{x}{2} + \frac{x^2}{6} + \dots \right)^{3/2}} = \frac{1}{(1)^{3/2}} = 1$$

Hence, the correct answer is (b).

5. (B) **Condition of collinearity**

$$\Delta = \begin{vmatrix} p+1 & 1 & 1 \\ 2p+1 & 3 & 1 \\ 2p+2 & 2p & 1 \end{vmatrix} = 0$$

$$\begin{aligned} \Rightarrow (p+1)(3-2p) - 1(2p+1-2p-2) \\ + 1(4p^2+2p-6p-6) &= 0 \\ \Rightarrow -2p^2+p+3+1+4p^2-4p-6 &= 0 \\ \Rightarrow 2p^2-3p-2 &= 0 \\ \Rightarrow 2p^2-4p+p-2 &= 0 \\ \Rightarrow p = 2, \frac{-1}{2} \end{aligned}$$

6. (B) When $x = 2, y = 4$ we have $4x + 3y = 4(2) + 3(4) = 20$.
 $\therefore (2, 4) \in R$.

7. (D) $\frac{2x+3}{5} < \frac{4x-1}{2}$
 $\Rightarrow 2(2x+3) < 5(4x-1)$
 [Multiplying both sides by 10.]
 We have: $a > b \Rightarrow am > bm \forall m > 0$
 $\Rightarrow 4x + 6 < 20x - 5$
 $\Rightarrow 0 < 16x - 11$
 [Adding $(-4x - 6)$ on both sides.]
 We have: $a > b \Rightarrow a + m > b + m \forall m \in \mathbb{R}$
 $\Rightarrow 11 < 16x$ [Adding 11 on both sides]
 $\Rightarrow \frac{11}{16} < x$ [Multiplying $\frac{1}{16}$ on both sides]
 $\therefore x \in \left[\frac{11}{16}, \infty \right)$

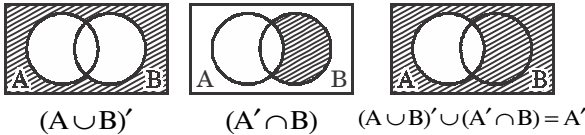
8. (A) $\tan(\pi \cos \theta)$
 $= \cot(\pi \sin \theta) = \tan\left(\pm \frac{\pi}{2} - \pi \sin \theta\right)$
 $\Rightarrow \pi \cos \theta = \pm \frac{\pi}{2} - \pi \sin \theta$
 $\Rightarrow \cos \theta + \sin \theta = \pm \frac{1}{2}$
 $\Rightarrow \sqrt{2} \cos\left(\theta - \frac{\pi}{4}\right) = \pm \frac{1}{2}$
 $\Rightarrow \cos\left(\theta - \frac{\pi}{4}\right) = \pm \frac{1}{2\sqrt{2}}$

9. (A) $\sum_{k=1}^{4n+7} i^k = i^1 + i^2 + i^3 + i^4 + i^5 + i^6 + i^7 + i^8 + \dots + i^{4n+1} + i^{4n+2} + i^{4n+3} + i^{4n+4} + i^{4n+5} + i^{4n+6} + i^{4n+7}$
 $= 0 + 0 + \dots + 0 + i - 1 - i = -1.$

10. (B) The desired point is the foot of the perpendicular from the origin on the line $3x - 4y = 25$.
 The equation of a line passing through the origin and perpendicular to $3x - 4y = 25$ is $4x + 3y = 0$.
 Solving these two equations we get $x = 3$, $y = -4$.
 Hence, the nearest point on the line from the origin is $(3, -4)$.

11. (B) Let a, b, c be the three sides of the triangle ABC , such that $a < b < c$ and a, b, c are in A.P. with common difference d .
 Then, $d > 0$ since a, b, c is an increasing A.P.
 $\therefore$ We have : $a = b - d$ and $c = b + d$
 Let A, B, C be the angles opposite to the sides a, b, c respectively.
 Then, $A < B < C$.
 [Side opposite the largest angle is the longest]
 We have: $C = 2A$ (given)
 And so, $B = \pi - (A + C) = \pi - (A + 2A) = \pi - 3A$.
 Now, by sine Formula, we have:
 $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
 $\Rightarrow \frac{b-d}{\sin A} = \frac{b}{\sin(\pi-3A)} = \frac{b+d}{\sin 2A}$
 $\Rightarrow \frac{b-d}{\sin A} = \frac{b}{\sin 3A} = \frac{b+d}{\sin 2A}$
 $\Rightarrow \frac{b}{b-d} = \frac{\sin 3A}{\sin A}$ and $\frac{b+d}{b-d} = \frac{\sin 2A}{\sin A}$
 $\Rightarrow \frac{b}{b-d} = 3 - 4\sin^2 A$ and $\frac{b+d}{b-d} = 2\cos A$
 [Using $\sin 3A = 3\sin A - 4\sin^3 A$]
 $\Rightarrow \frac{b}{b-d} = 3 - 4\sin^2 A$ and $\frac{b+d}{b-d} = 2\cos A$
 $[3 - 4\sin^2 A = 3 - 4(1 - \cos^2 A) = 4\cos^2 A - 1]$
 $\Rightarrow \frac{b}{b-d} = \left(\frac{b+d}{b-d}\right)^2 - 1$
 $\Rightarrow \frac{b}{b-d} = \frac{(b+d)^2 - (b-d)^2}{(b-d)^2} = \frac{4bd}{(b-d)^2}$
 $\Rightarrow 4d = b - d \Rightarrow b = 5d$.
 $\therefore$ The sides of the triangle are
 $a = b - d = 5d - d = 4d$, $b = 5d$ and
 $c = b + d = 5d + d = 6d$.
 Hence, the sides are in the ratio $4 : 5 : 6$

12. (C) $(A \cup B)' \cup (A' \cap B) = A'$ as shown in the following venn-diagrams:



13. (D) Number of ways to enter the palace = 7.
Number of ways to come out of the palace = 12.

∴ By the fundamental principle of counting:

Total number of ways in which a person can enter and come out of the palace = $7 \times 12 = 84$.

14. (C) Putting $x = \cos \theta$, we get

$$\begin{aligned} & \frac{d}{dx} \left\{ \sin^2 \left(\cot^{-1} \sqrt{\frac{1+x}{1-x}} \right) \right\} \\ &= \frac{d}{dx} \left\{ \sin^2 \left[\cot^{-1} \cot \frac{\theta}{2} \right] \right\} = \frac{d}{dx} \left(\sin^2 \frac{\theta}{2} \right) \\ &= \frac{d}{dx} \left[\frac{1}{2} \left(2 \sin^2 \frac{\theta}{2} \right) \right] = \frac{d}{dx} \left[\frac{(1 - \cos \theta)}{2} \right] \\ &= \frac{d}{dx} \left\{ \frac{1}{2} (1 - x) \right\} = -\frac{1}{2} \end{aligned}$$

∴ The correct answer is (c)

15. (B) The affixes of the points P, Q, R and S are $z_1 = 4 + i$, $z_2 = 1 + 6i$, $z_3 = -4 + 3i$ and $z_4 = -1 - 2i$ respectively.

$$PQ = |z_2 - z_1| = \sqrt{(1-4)^2 + (6-1)^2} = \sqrt{34};$$

$$QR = |z_3 - z_2| = \sqrt{(-4-1)^2 + (3-6)^2} = \sqrt{34};$$

$$RS = |z_4 - z_3| = \sqrt{(-1+4)^2 + (-2-3)^2} = \sqrt{34};$$

$$SP = |z_1 - z_4| = \sqrt{(4+1)^2 + (1+2)^2} = \sqrt{34};$$

$$\text{Also, } PR = |z_3 - z_1| = \sqrt{(-4-4)^2 + (3-1)^2} = \sqrt{68}$$

$$\text{and } QS = |z_4 - z_2| = \sqrt{(-1-1)^2 + (-2-6)^2} = \sqrt{68}.$$

Thus, all the four sides of the quadrilateral PQRS are equal i.e., $PQ = QR = RS = SP = \sqrt{34}$ and both the diagonals are equal i.e., $PR = QS$. Hence, PQRS is a square.

16. (A) Let x_1 and x_2 be the two roots of the equation

$$x^2 - 2ax + a^2 = 0$$

Then, $x_1 + x_2 = 2a$

$$\Rightarrow \frac{x_1 + x_2}{2} = a$$

i.e., Arithmetic mean of the roots, $A = a$.

Also, $x_1 x_2 = a^2$ [Product of the roots]

$$\Rightarrow \sqrt{x_1 x_2} = a$$

i.e., Geometric Mean of the roots, $G = a$.

Clearly, $A = G$.

17. (B) By Binomial Expansion:

$$(1+x)^{20} = {}^{20}C_0 + {}^{20}C_1 x + {}^{20}C_2 x^2 + \dots + {}^{20}C_{20} x^{20}$$

The general term in this expansion is given by

$$t_{r+1} = {}^{20}C_r x^r. \text{ Its coefficient is } {}^{20}C_r.$$

∴ Coefficient of $(m+1)$ th term = coefficient of $(m+3)$ th term

$$\Rightarrow {}^{20}C_m = {}^{20}C_{m+2} \Rightarrow {}^{20}C_{20-m} = {}^{20}C_{m+2} \quad [\because {}^{20}C_m = {}^{20}C_{20-m}]$$

$$\Rightarrow 20 - m = m + 2 \Rightarrow 2m = 20 - 2 \Rightarrow m = 9.$$

18. (D)

X = Set of all positive divisors of 400

$$= \{1, 2, 4, 5, 8, 10, 16, 20, 25, 40, 50, 80, 100, 200, 400\}$$

Y = Set of all positive divisors of 1000

$$= \{1, 2, 4, 5, 8, 10, 20, 25, 40, 50, 100, 125, 200, 250, 500, 1000\}$$

We have : $X \cap Y = \{1, 2, 4, 5, 8, 10, 20, 25, 40, 50, 100, 200\}$

Hence, $n(X \cap Y) = 12$.

19. (B)

We have $\tan A = a/b$ and $\tan B = b/a$, so

$$\text{that } \tan A + \tan B = \frac{a}{b} + \frac{b}{a}$$

$$= \frac{a^2 + b^2}{ab} = \frac{c^2}{ab}$$

20. (D) a, b, c are in A.P. $\Rightarrow b = \frac{a+c}{2}$ (i)

Now, $(a + 2b - c)(2b + c - a)(a + 2b + c)$

$$= \left\{ a + 2 \left(\frac{a+c}{2} \right) - c \right\} \left\{ 2 \left(\frac{a+c}{2} \right) + c - a \right\} \{ 2b + 2b \}$$

[Using (i)], we get

$$= (2a)(2c)(4b) = 16abc.$$

$$\begin{aligned}
 21. \quad (C) \quad & \lim_{x \rightarrow 2} \frac{xf(2) - 2f(x)}{x-2} \left[\text{Form } \frac{0}{0} \right] \\
 &= \lim_{x \rightarrow 2} \frac{xf(2) - 2f'(x)}{1} \\
 &= \{f(2) - 2f'(2)\} = (4 - 2 \times 4) = -4
 \end{aligned}$$

$\therefore$ The correct answer is (c).

$$\begin{aligned}
 22. \quad (C) \quad & |(1-i)(1+2i)(2-3i)| \\
 &= |1-i| \cdot |1+2i| \cdot |2-3i|
 \end{aligned}$$

$$\begin{aligned}
 [\square \quad |z_1 z_2 \dots z_n| &= |z_1| \cdot |z_2| \dots |z_n|] \\
 \left\{ \sqrt{1^2 + (-1)^2} \right\} \cdot \left\{ \sqrt{1^2 + 2^2} \right\} \cdot \left\{ \sqrt{2^2 + (-3)^2} \right\} \\
 &= \sqrt{2} \cdot \sqrt{5} \cdot \sqrt{13} = \sqrt{130}
 \end{aligned}$$

23. (B) Total number of ways of selecting 6 out of 10 friends $= {}^{10}C_6 = 210$.

Number of ways of selecting 6 out of 10 friends when 2 particular friends are to be always included (together) in each selection $= {}^{10-2}C_{6-2} = {}^8C_4 = 70$.

$\therefore$ Number of ways of selecting 6 out of 10 friends when 2 particular friends are not together in any selection

$$= 210 - 70 = 140.$$

$$24. \quad (A) \quad \text{Here } m_1 = 2 - \sqrt{3} \text{ and } m_2 = 2 + \sqrt{3}$$

If θ is the angle between lines, then

$$\tan \theta = \left[\frac{m_1 - m_2}{1 + m_1 m_2} \right]$$

$$\Rightarrow \tan \theta = \left[\frac{2 - \sqrt{3} - 2 - \sqrt{3}}{1 + (2 - \sqrt{3})(2 + \sqrt{3})} \right]$$

$$\Rightarrow \tan \theta = \frac{2\sqrt{3}}{1 + (4 - 3)}$$

$$\Rightarrow \tan \theta = \sqrt{3}$$

$$\therefore \theta = 60^\circ$$

25. (C) Let d be the common difference of the A.P. a, b, c .

Then, $b - a = d$ and $c - b = d$.

$\therefore a = b - d$ and $c = b + d$.

Now, $abc = 4$ (given)

$$\Rightarrow (b-d)b(b+d) = 4 \Rightarrow b(b^2 - d^2) = 4$$

..... (i)

But $b^2 - d^2 < b^2$ [Since both b^2 and d^2 are positive]

$$\Rightarrow b(b^2 - d^2) < b.b^2 \quad [\square \text{ be is positive}]$$

$$\Rightarrow 4 < b^3 \Rightarrow 2^2 < b^3 \Rightarrow b > 2^{2/3} \quad [\text{Using (i)}]$$

Thus, the minimum possible value of b is $2^{2/3}$.

26. (C) The sum of the coefficients in the expansion of $(x+y)^n = 4096$

$$\Rightarrow {}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_n = 4096$$

$$\Rightarrow 2^n = 4096 = 2^{12} \Rightarrow n = 12.$$

Since n is even, the greatest coefficient is the coefficient of the middle term i.e., of

the $\left(\frac{n}{2} + 1\right)$ th term i.e., of the 7th term.

Thus, the greatest coefficient in the expansion is

$${}^nC_{n/2} = {}^{12}C_6 = \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7}{6 \times 5 \times 4 \times 3 \times 2 \times 1} = 924.$$

27. (C) The number of proper subsets of a set having n elements is 2^{n-1} .

$\therefore$ The number of proper subsets of $\{a, b, c\}$ i.e. of a set containing 3 elements is $2^3 - 1$ i.e. 7.

28. (A) Let $P(h, 0)$ divides the join of $A(2, -3)$ and $B(5, 6)$ in the ratio of $\lambda : 1$, such that the co-ordinates of point P are

$$\left(\frac{5\lambda + 2}{\lambda + 1}, \frac{6\lambda - 3}{\lambda + 1} \right)$$

Since, the point P lies on the x -axis, therefore, the y -part (ordinate) must be zero.

$$\Rightarrow \frac{6\lambda - 3}{\lambda + 1} = 0$$

$$\Rightarrow \lambda = \frac{1}{2}$$

The ratio in which P on x -axis divided by join of AB is $1 : 2$

29. (C) When $n = 1$, we have :

$$p^{n+1} + (p+1)^{2n-1} = p^2 + (p+1)^1 = P^2 + p + 1, \text{ which is divisible by } p^2 + p + 1.$$

[Note : $p^2 + p + 1$ is not divisible by any of the remaining alternatives $p - 1, p^2 + 1$ or $p^3 - 1$.]

We shall show that $p^{n+1} + (p+1)^{2n-1}$ is divisible by $p^2 + p + 1$ for all natural numbers n .

Let $P(n) : p^{n+1} + (p+1)^{2n-1}$ is divisible by $p^2 + p + 1$.

Then, $P(1)$ is true. [Proved above]

Let $P(m)$ be true.

Then, $p^{m+1} + (p+1)^{2m-1}$ is divisible by $p^2 + p + 1$.

Let $p^{m+1} + (p+1)^{2m-1} = k(p^2 + p + 1)$
where $k \in \mathbb{N}$.

$$\Rightarrow p^{m+1} = k(p^2 + p + 1) - (p+1)^{2m-1}$$

$$\begin{aligned} \text{Now, } p^{(m+1)+1} + (p+1)^{2(m+1)-1} \\ &= p^{m+2} + (p+1)^{2m+1} = p \cdot p^{m+1} + (p+1)^{2m+1} \\ &= p \{k(p^2 + p + 1) - (p+1)^{2m-1}\} + (p+1)^{2m+1} \\ &= kp(p^2 + p + 1) - p(p+1)^{2m-1} + (p+1)^{2m+1} \\ &= kp(p^2 + p + 1) + (p+1)^{2m-1} \{-p + (p+1)^2\} \\ &= kp(p^2 + p + 1) + (p+1)^{2m-1}(p^2 + p + 1) \\ &= (p^2 + p + 1) \{kp + (p+1)^{2m-1}\}, \text{ which is divisible by } p^2 + p + 1. \end{aligned}$$

$\therefore P(m+1)$ is true whenever $P(m)$ is true.

Thus, $P(n)$ is true for all $n \in \mathbb{N}$.

Hence, $p^{n+1} + (p+1)^{2n-1}$ is divisible by $p^2 + p + 1$, for all $n \in \mathbb{N}$.

$$30. (C) \quad \frac{{}^2C_2}{{}^2C_4} = \frac{2}{1} \Rightarrow \frac{n!}{2!(n-2)!} \times \frac{4!(n-4)!}{n!} = \frac{2}{1}$$

$$\Rightarrow \frac{4 \times 3}{(n-2)(n-3)} = \frac{2}{1}$$

$$\Rightarrow n^2 - 5n + 6 = 6$$

$$\Rightarrow n^2 - 5n = 0$$

$$\Rightarrow n(n-5) = 0$$

$$\Rightarrow n = 0 \text{ or } n = 5$$

$\Rightarrow n = 5$ $\square$ nC_2 and nC_4 are not defined for $n = 0$

$$31. (C) \quad y = \log_e(1+x)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{(1+x)}$$

$$\Rightarrow \left. \frac{dy}{dx} \right|_{(x=1)} = \frac{1}{2}$$

$\therefore$ The correct answer is (C)

$$32. (B) \quad f(x) = \frac{x-1}{x+1} \Rightarrow (x+1)f(x) = x-1$$

$$\Rightarrow x\{f(x) - 1\} = -\{1 + f(x)\} \Rightarrow x = \frac{1+f(x)}{1-f(x)}$$

Now, $f(2x) =$

$$\frac{2x-1}{2x+1} = \frac{2\left\{\frac{1+f(x)}{1-f(x)}\right\}-1}{2\left\{\frac{1+f(x)}{1-f(x)}\right\}+1} = \frac{1+3f(x)}{3+f(x)} = \frac{3f(x)+1}{f(x)+3}$$

33. (C) The given equation can be written as

$$\frac{1+\sin x}{\cos x} = 2 \cos x$$

$$\Rightarrow 1 + \sin x = 2 \cos^2 x = 2(1 - \sin^2 x)$$

$$\Rightarrow 2 \sin^2 x + \sin x - 1 = 0$$

$$\Rightarrow (1 + \sin x)(2 \sin x - 1) = 0$$

$$\Rightarrow \sin x = -1 \text{ or } 1/2$$

Now $\sin x = -1 \Rightarrow x = 3\pi/2$ for which the given equation is not meaningful and $\sin x = 1/2 \Rightarrow x = \pi/6 \text{ or } 5\pi/6$.

The required number of solutions is 2.

$$34. (B) \quad \text{Let } y = \lim_{x \rightarrow 0} \left(\frac{a^x + b^x}{2} \right)^{1/x}, \text{ Then,}$$

$$\log y = \lim_{x \rightarrow 0} \frac{1}{x} \log \left(\frac{a^x + b^x}{2} \right)$$

$$= \lim_{x \rightarrow 0} \frac{\log(a^x + b^x) - \log 2}{x} \quad \left[\text{form } \frac{0}{0} \right]$$

$$= \lim_{x \rightarrow 0} \frac{1}{(a^x + b^x)} (a^x \log a + b^x \log b)$$

$$= \frac{1}{2} (\log a + \log b) = \log \sqrt{ab}$$

$$\therefore y = e^{\log \sqrt{ab}} = \sqrt{ab}$$

Hence, the correct answer is (b).

35. (C) The two digit natural numbers which leave a remainder 5 when divided by 7 are 12, 19, 26, ..., 89, 96

Let $S = 12 + 19 + 26 + \dots + 89 + 96$

Let there be n terms in the A.P. 12, 19, 26, ..., 89, 96.

The first term of this A.P. is $a = 12$ and its common difference $d = 19 - 12 = 7$.

Then the n th term $t_n = 96 \Rightarrow a + (n-1)d = 96$

$$\Rightarrow 12 + (n-1)(7) = 96 \Rightarrow 7n - 7 = 84 \Rightarrow n = 13.$$

Thus, there are 13 terms in the A.P. 12, 19, 26, ..., 89, 96.

$$\therefore S = \frac{n}{2} (a + l) = \frac{1}{2} (12 + 96) = 702.$$

36. (D)

The given expression is equal to

$$(\sin 47^\circ + \sin 61^\circ) - (\sin 11^\circ - \sin 25^\circ)$$

$$= 2 \sin 54^\circ \cos 7^\circ - 2 \sin 18^\circ \cos 7^\circ$$

$$= 2 \cos 7^\circ (\sin 54^\circ - \sin 18^\circ)$$

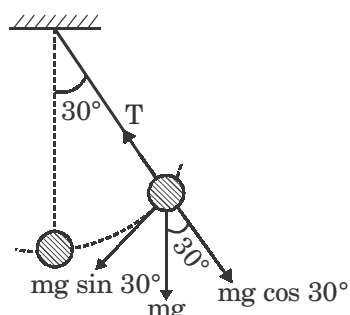
$$= 2 \cos 7^\circ \left[\frac{\sqrt{5}+1}{4} - \frac{\sqrt{5}-1}{4} \right] = \cos 7^\circ$$

37. (B) $\frac{2}{5} + \frac{3}{5^2} + \frac{2}{5^3} + \frac{3}{5^4} + \dots$
- $$\left(\frac{2}{5} + \frac{2}{5^3} + \dots\right) + \left(\frac{3}{5^2} + \frac{3}{5^4} + \dots\right)$$
- $$= \frac{\left(\frac{2}{5}\right)}{\left(1 - \frac{1}{5^2}\right)} + \frac{\left(\frac{3}{5^2}\right)}{\left(1 - \frac{1}{5^2}\right)} = \left(\frac{2}{5} \cdot \frac{25}{24}\right) + \left(\frac{3}{25} \cdot \frac{25}{24}\right)$$
- $$= \frac{10}{24} + \frac{3}{24} = \frac{13}{24}$$
38. (C) $A = \{1, 2\}, C = \{\{1\}, \{2\}\}$
Clearly, $A \in C$.
39. (C) $3 \leq 3t - 18 \leq 18$ (i) (given)
 $\Rightarrow 21 \leq 3t \leq 36$ (ii)
 $[a > b \Rightarrow a + m > b + m \forall m \in \mathbb{R}]$
 $\therefore$ Alternative (B) is not true.
 $\Rightarrow 7 \leq t \leq 12$ (iii)
- $$\left[\square a > b \Rightarrow \frac{a}{m} > \frac{b}{m} \forall m > 0 \right]$$
- $\therefore$
- Alternative (A) is not true.
-
- $\Rightarrow 8 \leq t + 1 \leq 13$
- (iv)
-
- [Adding 1 on both sides]
-
- $\therefore$
- Alternative (C) is true.
-
- Again, from (iii) we have:
- $7 \leq t \leq 12$
-
- $\Rightarrow 14 \leq 2t \leq 24$
- [M u l t i p l y i n g both sides by 2]
-
- $\Rightarrow 15 \leq 2t + 1 \leq 25$
- [Adding 1 on both sides]
-
- $\therefore$
- Alternative (D) is not true.
40. (C) We have: $\alpha - i\beta = \left(\frac{3 - 4i x}{3 + 4i x}\right)$
- $$\Rightarrow |\alpha - i\beta| = \left|\frac{3 - 4i x}{3 + 4i x}\right| = \frac{|3 - 4i x|}{|3 + 4i x|} \quad \left[\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}\right]$$
- $$\Rightarrow |\alpha - i\beta|^2 = \frac{|3 - 4i x|^2}{|3 + 4i x|^2}$$
- $$\Rightarrow \alpha^2 + \beta^2 = \frac{3^2 + (-4x)^2}{3^2 + (4x)^2} = \frac{9 + 16x^2}{9 + 16x^2} = 1.$$

Physics

41. (B) Here, $m_1 = 10 \text{ kg}$, $m_2 = 5 \text{ kg}$, $v_1 = ?$, $v_2 = 0$, $v = 4 \text{ m/s}$
Applying the principle of conservation of linear momentum,
 $m_1 v_1 + m_2 v_2 = (m_1 + m_2) v$
 $10v_1 + 5 \times 0 = (10 + 5)4 = 60$
 $v_1 = \frac{60}{10} = 6 \text{ m s}^{-1}$
42. (C) $d = 0.005 \text{ mm} = 0.005 \times 10^{-3} \text{ m}$,
 $R = 10/2 = 5 \text{ cm} = 0.05 \text{ m}$,
 $\sigma = 0.072 \text{ Nm}^{-1}$
Force required
 $= \frac{2\sigma}{d} \times \pi R^2 = \frac{2 \times 0.072 \times 3.14 \times (0.05)^2}{0.005 \times 10^{-3}}$
 $= 226 \text{ N}$
43. (C) Here $\alpha = (10/2) \text{ rad s}^{-2}$
 $I = MR^2 = \frac{1}{2} \times (0.2)^2 = 0.02 \text{ kg ms}^2$.
Here $\tau = I\alpha = 5 \times 0.02 \text{ Nm} = 0.10 \text{ Nm}$.
44. (D) $\eta = \frac{W}{Q_1}$ $W = \eta Q_1 = \frac{1}{3} \times 1000 \text{ cal}$.
 $= \frac{1000}{3} \times 4.2 \text{ joule} = 1400 \text{ J}$.
45. (B) Mean value of refractive index.
 $n_m = \frac{1.49 + 1.50 + 1.52 + 1.54 + 1.48}{5}$
 $= 1.506 = 1.51$
46. (C) Spring constant, $k = F/x$.
Workdone $= \frac{1}{2} k x^2 = \frac{1}{2} \times \frac{4 \times 9.8}{0.02} \times (0.05)^2$
 $= 2.45 \text{ J}$
47. (A) $\rho = 0.09 \text{ kg m}^{-3}$, $v_{\text{rms}} = 1.84 \times 10^3 \text{ m s}^{-1}$
Pressure $= P = \frac{1}{3} \rho v_{\text{rms}}^2$
 $P = \frac{1}{3} \times 0.09 \times (1.84 \times 10^3)^2 = 1.015 \times 10^5 \text{ N/m}^2$

48. (D) At height h , $\frac{g'}{g} = 1 - \frac{2h}{R} = \frac{90}{100}$
- or $\frac{2h}{R} = 1 - \frac{90}{100} = \frac{10}{100} = \frac{1}{10}$
- or $R = 20h = 20 \times 320 = 6400 \text{ km}$
- At depth d ,
- $\frac{g'}{g} = 1 - \frac{d}{R} = \frac{95}{100}$
- or $\frac{d}{R} = 1 - \frac{95}{100} = \frac{5}{100} = \frac{1}{20}$
- or $d = \frac{R}{20} = \frac{6400}{20} = 320 \text{ km}$
49. (C) Hydrogen atom is assumed to be at the origin
- $x_1 = 0, m_1 = m, x_2 = 1.27 \text{ \AA}$
- $= 1.27 \times 10^{-10} \text{ m}$
- $m_2 = 35.5 \text{ m}$
- The position of centre of mass
- $$x = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$
- $$= \frac{m \times 0 + 35.5 \text{ m} \times 1.27 \times 10^{-10}}{m + 35.5 \text{ m}}$$
- $$= \frac{45.08 \times 10^{-10} \text{ m}}{36.5}$$
- $$= 1.24 \times 10^{-10} \text{ m}$$
50. (C) Power = $\vec{F} \cdot \vec{v} = Fv$
- $$F = v \left(\frac{dm}{dt} \right)$$
- $$= v \left\{ \frac{d(\rho \times \text{volume})}{dt} \right\} \quad (\rho = \text{density})$$
- $$= \rho v \left\{ \frac{d(\text{volume})}{dt} \right\} = \rho v (Av) = \rho A v^2$$
- $\therefore$ power $P = \rho A v^3$
- or $P \propto v^3$
51. (D) Sudden compression causes heating and rise in temp. With the passage of time, temp, falls and hence pressure also decreases.

52. (D) $A + B = 17$ (i)
- $A - B = 7$ (ii)
- On solving $A = 12$ units and $B = 5$ units
- $R = \sqrt{A^2 + B^2} = \sqrt{12^2 + 5^2} = 13$ units.
53. (B) Impulse = change in linear momentum
- $= m v - m u = 0.1 \times 40 - 0.1 \times (-30)$
54. (B) Sum of P.E. and K.E. of the ball at A = Sum of P.E. and K.E. of the ball at B. At the point A, the ball is at rest, so K.E. = 0.
- At B the speed of the ball is v . At B, $h = 0$
- So, P.E. = 0
- (P.E. + K.E.) at A = (P.E. + K.E.) at B
- $$mgh + 0 = 0 + \frac{1}{2} mv^2$$
- $$v\sqrt{2gh}$$
- $$= \sqrt{2 \times 9.8 \times 10}$$
- $$= 14 \text{ m/s}$$
55. (B) Rate of change of speed
- $$\frac{dv}{dt} = \text{tangential acceleration}$$
- $$= \frac{\text{tangential force}}{\text{mass}}$$
- $$= \frac{mg \sin 30^\circ}{m} = g \sin 30^\circ$$
- $$= 10 \left(\frac{1}{2} \right) \text{ m/s}^2 = 5 \text{ m/s}^2$$
- 
56. (B) Workdone = $\frac{1}{2} \times Y \times (\text{strain})^2 \times \text{volume}$
- For a given strain, workdone $\propto Y$.
- As Y for copper is less than that of steel, so less work is done on copper spring than that on steel spring.

57. (A) As on the surface of planet $g = (GM/R^2)$

$$\frac{g_M}{g_E} = \frac{M_M}{M_E} \times \left(\frac{R_E}{R_M}\right)^2 = \frac{1}{80} \times (4)^2 = \frac{1}{5}$$

$$\therefore g_M = \frac{g}{5} = \frac{9.8}{5} = 1.96 \text{ m/s}^2$$

Further more as escape velocity

$$v_e = \sqrt{(2GM/R)}$$

$$\frac{v_M}{v_E} = \sqrt{\frac{M_M}{M_E} \times \frac{R_E}{R_M}} = \sqrt{\frac{1}{80} \times 4} = \frac{1}{\sqrt{20}}$$

$$\text{i.e., } v_M = (v_E / \sqrt{20}) = (11.2/4.47) = 2.5 \text{ km/s.}$$

58. (A) Mass = $m = 700 \text{ gm} = 0.7 \text{ kg}$

Radius of gyration = $K = 20 \text{ cm} = 0.2 \text{ m}$

Moment of inertia = $I = mK^2 = 0.7 \times 0.2 \times 0.2 = 0.028 \text{ kg m}^2$.

59. (C) Let a be the acceleration of sprinter for time $t = 4 \text{ s}$.

$$\text{Distance travelled in } 4 \text{ s} = \frac{1}{2} at^2 = \frac{1}{2} \times a \times 4^2 = 8a$$

Velocity of sprinter at time 4 sec is

$$v = at = a \times 4 = 4a$$

Distance covered by sprinter with uniform velocity in time 4 sec to 9 sec ($= 5 \text{ s}$) = $4a \times 5 = 20a$

As per question, $8a + 20a = 140$

$$\text{or } a = 5 \text{ m s}^{-2}$$

60. (D) $\frac{P_1}{P} = \frac{T_1}{T}$

$$\text{or } T_1 = P_1 \frac{T}{P} = \frac{2P \times 293}{P} = 586 \text{ K} = 313^\circ \text{C}$$

61. (B) Here, $m = 120 \text{ g} = 0.12 \text{ kg}$

$$\vec{v} = (2\hat{i} + 5\hat{j}) \text{ m/s}$$

$$v = |\vec{v}| = \sqrt{2^2 + 5^2} = \sqrt{29} \text{ m/s}$$

$$\text{KE} = \frac{1}{2} mv^2 = \frac{1}{2} \times 0.12 \times 29 = 1.74 \text{ joule.}$$

62. (A) Work done in a cyclic process = area between the cycle

$$= AB \times BC$$

$$= (2P - P) \times (2V - V)$$

$$= PV$$

63. (A) Given, $\frac{mg'}{mg} = \frac{30}{90}$ or $\frac{g'}{g} = \frac{1}{3}$

$$\text{Now } g' = g \frac{R^2}{(R+h)^2} \text{ or } g' = g \frac{R^2}{(R+h)^2} = \frac{1}{3}$$

$$\text{or } \frac{R}{R+h} = \frac{1}{\sqrt{3}} \text{ or } (R+h) = \sqrt{3} R$$

$$\text{or } h = (\sqrt{3} - 1) R = 0.73 R$$

64. (B) $\frac{4}{3} \pi R^3 = 10^6 \left(\frac{4}{3} \pi r^3 \right)$

$$\text{or } R = (10^2 r)$$

$$U_i = 10^6 (4 \pi r^2) T$$

$$U_f = (4 \pi R^2) T$$

$$\therefore \frac{U_f}{U_i} = \frac{1}{10^2}$$

65. (B) Thrust = normal force

$$\text{Mass ejected per sec} = 250 \times 10^{-3} \text{ kg s}^{-1}$$

$$\text{Change of velocity} = 8000 \text{ m s}^{-1}$$

$$F = \frac{m(v-u)}{t}$$

$$= \text{Mass per sec} \times \text{Change of velocity}$$

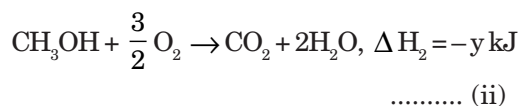
$$F = 250 \times 10^{-3} \times 8 \times 1000 = 2000 \text{ N}$$

Chemistry

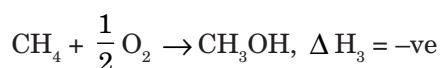
66. (B) $\begin{array}{ccccccc} {}^1\text{CH}_3 & - & {}^2\text{CH} & - & {}^3\text{CH} & - & {}^4\text{CH}_2 & - & {}^5\text{CH}_3 \\ & & | & & | & & & & \\ & & \text{H}_3\text{C} & & \text{CH}_2\text{CH}_3 & & & & \end{array}$
3-Ethyl-2-methylpentane

67. (D) O^{2-} ($8 + 2 = 10$) and Mg^{2+} ($12 - 2 = 10$) are isoelectronic with F^- ($9 + 1 = 10$)

68. (A) $\text{CH}_4 + 2\text{O}_2 \rightarrow \text{CO}_2 + 2\text{H}_2\text{O}$, $\Delta H_1 = -x \text{ kJ}$ (i)



Subtracting (ii) from (i), we get

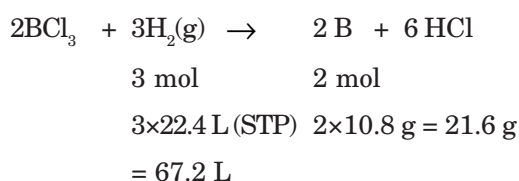


$$\text{i.e., } -x - (-y) = -ve$$

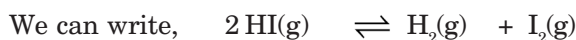
$$y - x = -ve.$$

Hence, $x > y$

69. (B)



70. (C)



Initial pressure, 0.2 atm, 0 0

Equilibrium partial pressure,

$$0.04 \text{ atm} \quad \frac{(0.2 - 0.04)}{2} \quad \frac{(0.2 - 0.04)}{2}$$

$$= 0.08 = 0.08$$

$$\text{Then, } K_p = \frac{[\text{H}_2][\text{I}_2]}{[\text{HI}]^2} = \frac{0.08 \times 0.08}{(0.04)^2} = 4$$

71. (D) Na_2CO_3 is completely dissociated to Na^+ and CO_3^{2-} ions thus giving high concentration of CO_3^{2-} ions which would precipitate Mg^{2+} as MgCO_3 .

72. (C) From the given data,

$$n(\text{H}_2) = \frac{0.5 \text{ g}}{2 \text{ g/mol}} = 0.25 \text{ mol}$$

$$n(\text{O}_2) = \frac{10 \text{ g}}{32 \text{ g/mol}} = 0.31 \text{ mol}$$

$$n(\text{H}_2) = \frac{15 \text{ L}}{22.4 \text{ L/mol}} = 0.67 \text{ mol}$$

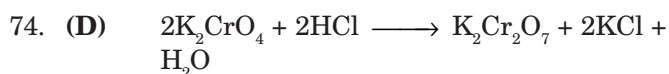
$$n(\text{N}_2) = \frac{5 \text{ L}}{22.4 \text{ L/mol}} = 0.22 \text{ mol}$$

Number of molecules $\propto$ No. of moles of the gas

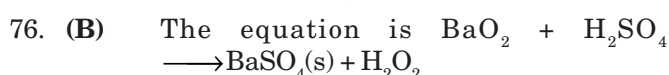
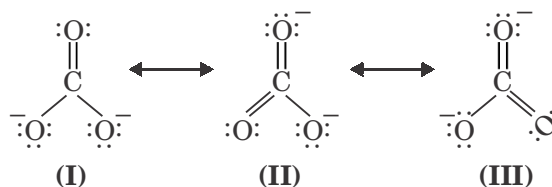
So, 15 L of H_2 gas at STP will contain the maximum number of molecules.

73. (A) $\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3}{4}, \left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{5}{36}, \left(\frac{1}{3^2} - \frac{1}{4^2}\right) = \frac{7}{144}$
and so on.

Thus, $1/\lambda$ decreases from transition $n = 2$ to $n = 1$, to $n = 3$ to $n = 2$ and so on or λ increases in the reverse order, i.e., λ is maximum for $n = 5$ to $n = 4$. As by de Broglie equation, $\lambda \propto 1/p$, therefore, momentum p will be maximum for the transition $n = 2$ to $n = 1$.



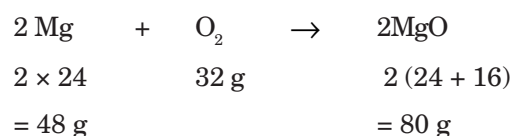
75. (B) According to the experimental findings, all carbon to oxygen bonds in CO_3^{2-} are equivalent. Therefore, the carbonate ion is best described as a resonance hybrid of the canonical forms I, II and III shown below.



The most electronegative atom in the products is oxygen. The oxidation state of O in H_2O_2 is -1 , whereas that is BaSO_4 is -2 .

77. (D) $\%P = \frac{62}{222} \times \frac{0.444}{0.31} \times 100 = 40.$

78. (D)



30 g Mg will react completely with $\text{O}_2 = \frac{32}{48} \times 30 = 20 \text{ g}$

$\therefore \text{O}_2 \text{ left} = 30 - 20 = 10 \text{ g}$

$\text{MgO formed} = \frac{80}{48} \times 30 = 50 \text{ g}$

79. (D) CCl_4 has tetrahedral, symmetrical structure, hence does not show dipole moment.

80. (C) Boron halides, BX_3 have only six electrons (three from B atom and three from three halogen atoms) around central B atom and thus B octet is not complete. Hence, boron halides behave as electron deficient compounds or Lewis acids.

81. (A)

	A	+	2B	$\rightleftharpoons$	C
Initial moles	1		0.5		
Moles at eqm.	0.9		0.3		0.1
Molar concs.	0.3		0.1		0.1/3

$$K = \frac{0.1/3}{0.3 \times (0.1)^2} = 11.11$$

82. (B) Here, IE_2 involves the removal of an electron from the inert gas configuration ($1s^2 2s^2 p^6$)
83. (C) $PbS + 4 H_2O_2 \rightarrow PbSO_4 \downarrow + 4H_2O (l)$
84. (C) The boiling points of benzene (353 K) and chloroform (334 K) differ by 19° and hence can be separated by distillation.
85. (B) Given
 Amount of the gas, $n = 4.0 \text{ mol}$
 Volume of the gas, $V = 5 \text{ dm}^3$
 Pressure of the gas, $P = 3.32 \text{ bar}$
 $R = 0.083 \text{ bar dm}^3 \text{ K}^{-1} \text{ mol}^{-1}$
 From the gas equation, $PV = nRT$
- $$T = \frac{PV}{nR} = \frac{3.32 \text{ bar} \times 5 \text{ dm}^3}{4.0 \text{ mol} \times 0.083 \text{ bar dm}^3 \text{ K}^{-1} \text{ mol}^{-1}} = 50 \text{ K}$$
86. (B) Eqm. const. is constant at constant temperature.
87. (A) $MgSO_4 \cdot 7 H_2O$. Because of smaller size, Mg^{2+} ions are extensively hydrated.
88. (C) The process (C) cannot go by itself after initiation.
89. (B) Graphite is thermodynamically, the most stable allotrope of carbon. That is why $\Delta_f H^\circ$ (graphite) is taken as zero.
- $\Delta_f H^\circ$ (diamond) = $+ 1.90 \text{ kJ mol}^{-1}$
- $\Delta_f H^\circ$ (fullerene) = $+ 38.1 \text{ kJ mol}^{-1}$
90. (B) Heterocyclic and aromatic, i.e., furan.

