THE ASSOCIATION OF MATHEMATICS TEACHERS OF INDIA

Screening Test - Ramanujan Contest

*(NMTC- at **INTER LEVEL** XI & XII Standards)*

Saturday, 22nd August 2015.

Note:

1) *Fill in the response sheet with your Name, Class, the institution through which you appear in the specified places.*
2) *Diagrams are only visual aids; they are not drawn to scale.*
3) *You are free to do rough work on separate sheets.*
4) *Duration of the test: 2 p.m. to 4. p.m.- 2 hours.*

PART – A

Note:

- *Only one of the choices A,B,C,D is correct for each question. Shade that alphabet of your choice in the response sheet. (If you have any doubt in the method of answering, seek the guidance of your supervisor).*
- *For each correct response you get 1 mark; **for each incorrect response you lose ½ mark.***

1. The radii of two circles are 5 cm and 2 cm. The length of the direct common tangent of the circles is 1.5 times the length of the transverse common tangent. The distance between the centres of the circles is (in cm)

   (A) 8 (B) 9 (C) 10 (D) 11

2. The geometric mean of two positive numbers is greater by 12 than the smaller number and the arithmetic mean of the same number is smaller by 24 than the larger number. the sum of the two numbers is

   (A) 56 (B) 29 (C) 72 (D) 60

3. The number of real solutions (x, y) of the equation $\left(16x^{200}+1\right)\left(y^{200}+1\right)=16(xy)^{100}$ is

   (A) 100 (B) 200 (C) 400 (D) 4

4. ABC is an isosceles triangle in which $AB = AC$. A circle is drawn passing through B and touching AC at its midpoint M. The circle cuts AB at P. The value of $\frac{BP}{AP}$ is

   (A) 1 (B) 2 (C) 3 (D) 4

5. Integers a, b, c, d satisfy $|ac+bd|=|ad+bc|=1$
   Then
   (A) $a=b=c=d$ is the only possibility.
   (B) $|a| = |b|$ and $|c| = |d|$ is the only possibility
   (C) $|a| = |b| = 1$ and $|c| = |d| = 1$
   (D) $|a| = |b| = 1$ or $|c| = |d| = 1$

6. Two bicyclists *Samrud* and *Saket* ride round a circular course of perimeter 18km in opposite directions. *Samrud* rides at 10km an hour and *Saket* at 14km an hour. The time after the start when they meet for the second time is

(A) 1 hour 15 minutes (B) 1 hour 30 minutes
(C) 1 hour 45 minutes (D) 2 hours

7. The angles of a triangle are in the ratio 2 : 3 : 7. The length of the smallest side is 2015 cm. the radius of the circum circle of the triangle (in cm) is

(A) 2015 (B) 4030 (C) $\frac{2015}{2}$ (D) 8045

8. The area under the curve $y=\frac{|x-3|+|x+1|}{|x+3|+|x-1|}$, x -axis and the ordinates at $x=-3$ and $x=1$ is

(A) 5 (B) 6 (C) 4 (D) 7

9. The number 27000001 has exactly four prime factors. The sum of these factors is

(A) 573 (B) 612 (C) 643 (D) 652

10. d is the common difference of the AP $\{a_n\}$. r is the common ratio of the GP $\{b_n\}$. r is a positive rational number less than 1. Given $a_1=d$, $b_1=d^2$ and $\frac{a_1^2+a_2^2+a_3^2}{b_1+b_2+b_3}$ is a positive integer, then the value of r is

(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $\frac{1}{4}$ (D) $\frac{1}{6}$

11. The value of $\frac{\sqrt{3}}{\sqrt{\sqrt{3}+1}-1}+\frac{\sqrt{3}}{\sqrt{\sqrt{3}+1}+1}$ is

(A) $2\sqrt{3}$ (B) $\sqrt{3}$ (C) 2 (D) $\frac{\sqrt{3}}{2}$

12. In a right triangle, the ratio of the circum radius to the inradius is 5:2. One acute angle of the triangle is

(A) $\tan^{-1}\left(\frac{3}{4}\right)$ (B) $\tan^{-1}\left(\frac{3}{5}\right)$ (C) $\tan^{-1}\left(\frac{4}{5}\right)$ (D) $\tan^{-1}\left(\frac{1}{2}\right)$

13. The number of real solutions of the equations $|x^2-2x|+y=1$ and $x^2+|y|=1$ is

(A) 1 (B) 2 (C) 3 (D) 4

14. In an isosceles triangle the altitude drawn to the base is $\frac{2}{3}$ times the radius of the circum circle. The base angle of the triangle is

(A) $\cos^{-1}\left(\frac{2}{3}\right)$ (B) $\cos^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (C) $2\cos^{-1}\sqrt{\frac{2}{3}}$ (D) $\cos^{-1}\sqrt{\frac{2}{3}}$

15. For real values of x and y, the minimum value of the expression $x^2+2xy+3y^2+2x+6y+4$

(A) 0 (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) 1

PART – B

Note:

- *Write the correct answer in the space provided in the response sheet.*
- *For each correct response you get I mark;* ***for each incorrect response you lose ¼ mark.***

16. In an infinite G.P of common ratio r ($r < 1$) every term is 4 times as large as the sum of all its successive terms. The value of r is _____.

17. The hypotenuse of a right isosceles triangle is 2015. The perimeter of the in circle is $\frac{2015\pi}{K}$. The value of K is _____.

18. If $n=\frac{\log_7 4(\log_7 5-\log_7 2)}{\log_7 25(\log_7 8-\log_7 4)}$ then the value of 5^n is _____.

19. If $\sec x+\tan x=\frac{22}{7}$, $\text{cosec}\, x+\cot x=\frac{m}{n}$ where $(m,n)=1$, then the value of $m+n$ is _____.

20. The circum circle of the quadrilateral $ABCD$ has a radius 2. AC, BD cut at E such that $AE=EC$. If $AB=\sqrt{2}AE$, $BD=2\sqrt{3}$, the area of the quadrilateral $ABCD$ is _____.

21. The smallest natural number 'n' which satisfies $12^{200} < n^{300}$ is _____.

22. If $\sqrt{x+1}+\sqrt{y}+\sqrt{z-4}=\frac{x+y+z}{2}$, then the numerical value of $x+y+z$ is _____.

23. $ABCDEFGH$ is a regular octagon. P is the point inside the octagon such that Δ ABP is equilateral. The measure of the angle APC is _____.

24. The sum of the infinite series

$$\frac{1}{2}+\frac{2}{8}+\frac{3}{16}+\frac{5}{32}+\frac{8}{64}+\frac{13}{128}+\frac{21}{256}+\frac{34}{512}+\ldots\ldots\ldots$$ is _____.

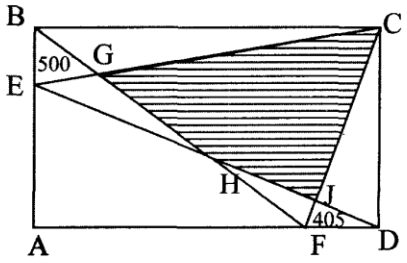
25. A seven digit number consists o the digits 0,1,2,3,4,6 and 8 each used once. The largest such number that is a multiple of 120 is _____.

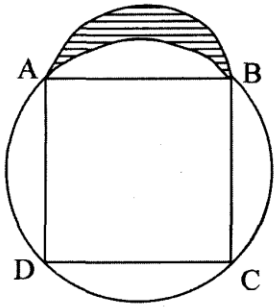
26. a,b,c,d,e are real numbers such that

$$a+4b+9c+16d+25e=1$$
$$4a+9b+16c+25d+36e=8$$
$$9a+16b+25c+36d+49e=23$$

The value of $a+b+c+d+e$ is _____.

27. In the adjoining figure $ABCD$ is a rectangle. Area of $\Delta\ BEG$ = 500. Area of $\Delta\ JFD$ = 405. Area of quadrilateral $EHFA$ = 1110. Area of the shaded region is _____.



28. 

$ABCD$ is a square inscribe in a circle of unit radius. On AB as diameter a semicircle is drawn. Area of the shaded region is _____.

29. The smallest number which is the sum of two different primes in two ways is _____.

30. The number less than 100 which increase by 20% when its digits are reversed is _____.

_____0_____